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LASER DOPPLER ANEMOMETER MEASUREMENT AND ANALYTICAL COMPARISON OF FLOW AROUND A CYLINDER AT LOW REYNOLDS NUMBER

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Monterey, California

THESIS

Laser Doppler Anemometer Measurement and Analytical Comparison of Flow Around a Cylinder at Low Reynolds Number

by

Terry Scott Wanner

March 1976

Thesis Advisor: D. J. Collins

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In this study, the flow characteristics of air around a circular cylinder were determined by two means: by analytical solution of the Navier-Stokes equations, and actual measurement of the flow itself. A DISA LD 5586 laser doppler anemometer system was employed to measure normal (v) and parallel (u) velocity components of a laminar flow regime about a three-sixteenths inch diameter cylinder mounted in a 32 x 45 inch wind tunnel (Reynolds number approximately 40).

A finite element grid was constructed using triangular elements which encompassed the upper half-cylinder. Flow measurements using the laser velocimeter were confined to this region of interest.

Theory was compared against experimental results, and the feasibility of the utilization of this particular type of laser anemometer was evaluated.

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by

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Submitted in partial fulfillment of the requirements for the degree of

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March 1976

I. ABSTRACT

In this study, the flow characteristics of air around a circular cylinder were determined by two means: by analytical solution of the Navier-Stokes equations, and actual measurement of the flow itself. A DISA LD 5586 laser doppler anemometer system was employed to measure normal (v) and parallel (u) velocity components of a laminar flow regime about a three-sixteenths inch diameter cylinder mounted in a 32 x 45 inch wind tunnel (Reynolds number approximately 40).

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II. DESCRIPTION OF LASER ANEMOMETER

A basic laser anemometer is comprised of four primary elements:

(1) a coherent light source

(2) optics

(3) signal converter (photomultiplier tube)

(4) signal processor/display

The principle of the laser anemometer is to create a grid pattern and measure the speed with which a particle in a fluid free stream passes through the grid. The following assumptions were taken into consideration:

(1) particles injected into the free stream are small enough to faithfully follow stream lines of the flow, thus by measuring particle characteristics, we are actually measuring the medium's characteristics. This is easily achieved by use of particles which are on the order of a few microns in diameter.

(2) particle generation results in a consistent particle size, or at least a majority of the particles are within a specified range of sizes. For example, DOP (di-2-ethylhexyl phthalate) particle generators produce particles with a mean diameter of 0.75 microns and burning turbine oil produces particles on the order of 4-6 microns (from electron microscope).

A laser is incorporated because of its coherency and power. The aforementioned grid is in reality the result of two coherent beams interfering with themselves (Fig. 1). The optics produce the beam pair by conventional splitting and focusing. Because the laser beam intensity is of a Gaussian distribution the grid created by constructive and destructive interference is not uniform over the entire focal volume. The fringes created on the outer portions of the horizontal axis are not perfectly formed because of slightly different intensities in that area. The main portion of the interference grid is uniform and, most importantly, the fringe spacing (D_f) is determined only by the laser wavelength (lambda) and the half angle of the beam pair convergence (theta/2).

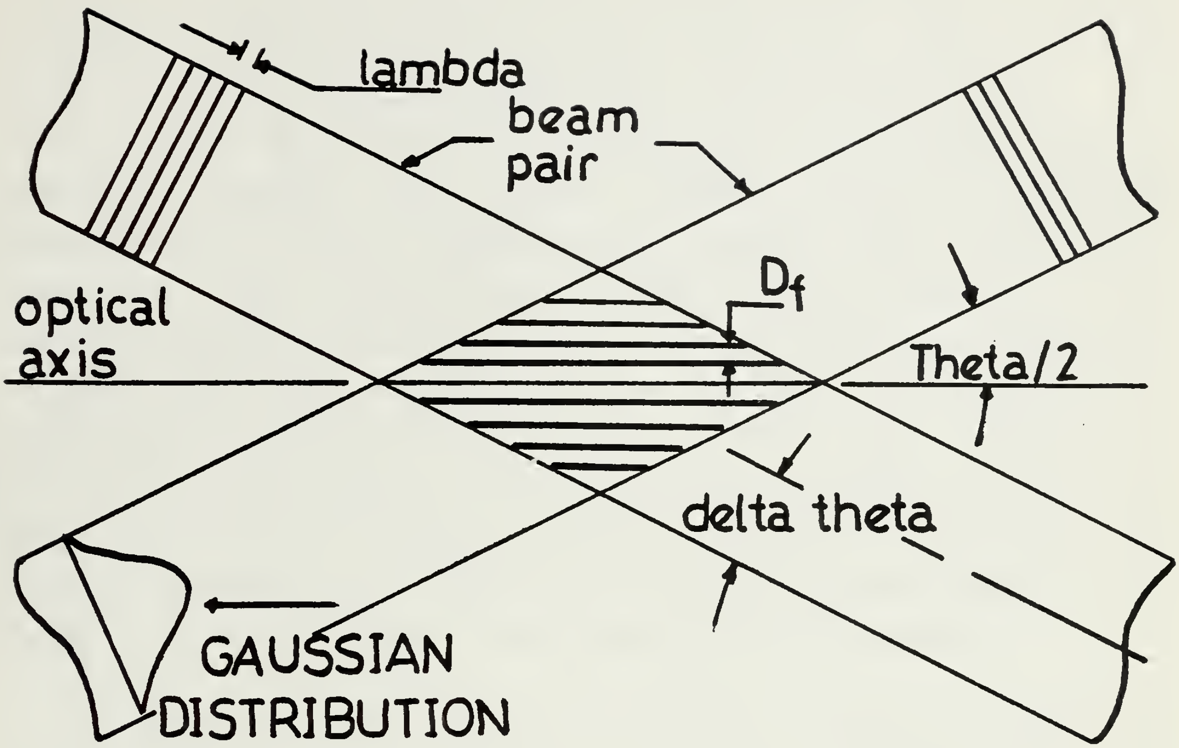
$$D_f = \text{lambda}/(2*\sin(\text{Theta}/2))$$

The number of fringes (N_f) may also be computed, provided the beam focusing convergence angle (delta theta) is known ,by the relation:

$$N_f = 8*\tan(\text{Theta}/2)/(\text{PI}*\text{Delta Theta})$$

The number of fringes is important only to ensure that the signal processing apparatus will receive enough information to make possible further data reduction. If an insufficient number of fringes exists, not enough signal will be available for velocity determination.

Solid particles with a mean diameter which is less than half the fringe spacing are introduced into the free stream



FRINGE PATTERN GEOMETRY

Figure 1

of the fluid to be measured. When the particle is small compared to the fringe spacing the scattered light exhibits essentially 100% modulation, while particles equal to the fringe spacing produce negligible modulation[Ref. 2]. The particles may be produced by atomization of water, dop-air reaction, burning oil, smoke bombs, or may be bought commercially, as e.g. polystyrene spheres and some solid chemicals such as zinc oxide. Choice of the particle to be used is normally dictated by the fringe spacing in the focal volume, and the nature of the medium to be measured. Regardless of choice, the particle must be small enough to follow the streamlines as closely as possible, because the laser anemometer actually measures the particle velocity and unless the particle's motion is a true representation of the fluid velocity, pertinent data may not be obtained.

Inside the measuring (or focal) volume, bands of darkness and brightness exist due to the interference of the two beams. Particles entering the focal volume will alternately scatter light as they pass through regions of constructive interference (typical focal volume dimensions are 300 microns wide by 4 millemeters long and contain approximately 64 fringes). A photomultiplier tube located on the optical axis and focused into the measuring volume receives this scattered light and converts what appears to be a modulated light signal into a series of voltage pulses. Each voltage pulse correlates to a particle crossing one fringe. Because the spacing of the fringes is known, the frequency of the voltage pulse train may be equated to the velocity of the particle by the following relationship:

$$f=V_o/D_f = 2*V_o*\sin(Theta/2)/lambda$$

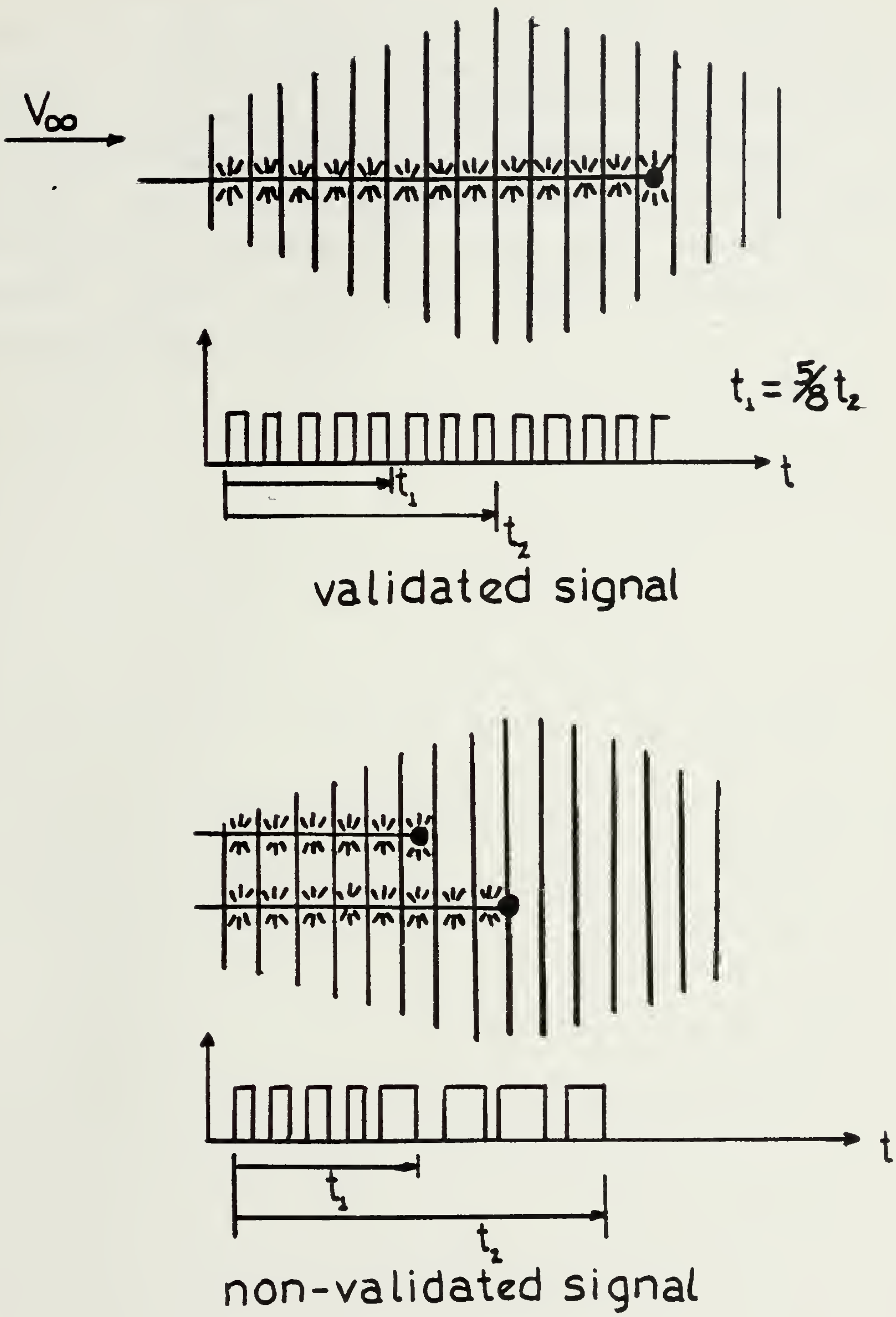
where D_f is the fringe spacing and V_0 is the velocity of the

particle. Now the processor is left with a fairly simple task of counting the frequency and multiplying it by a calibration factor, which is, in essence, a fringe spacing parameter to arrive at a velocity.

As long as we are assured that only one particle is in the measuring volume at one time, the apparatus will operate correctly, but this is an unrealistic assumption. In order to achieve such a situation, the seeding density of the particles would be exceptionally sparse and the resulting data rate would be so slow as to preclude timely data reduction. To overcome this obstacle, a validation circuit is incorporated into the counter/processor. When a particle crosses a fringe spacing, two counters are triggered. One measures the time duration for five fringe crossings and holds this information. The second counter continues to measure the time for a total of eight fringe crossings. The time of the first counter is then compared with that of the second counter, and if the first time is indeed five-eights of the second time (within specified limits, i.e., comp. accuracy) a validation gate is opened and the resultant frequency is then stored into an ensemble cell. The ensemble cell is adjustable to hold various numbers of counts and average them for the final velocity display. The final displayed velocity then is that component of fluid velocity which is perpendicular to the optical axis and contained in the same plane as the beam pair (Fig. 2).

The anemometer may be operated in three modes: reference beam, forward scatter, and back scatter. The last two modes are most frequently employed and are described here:

Forward scatter mode: in this mode, the photomultiplier tube is located on the optical axis and focused in a direction directly opposed to the direction of the emerging beam pairs. This mode offers the strongest attainable

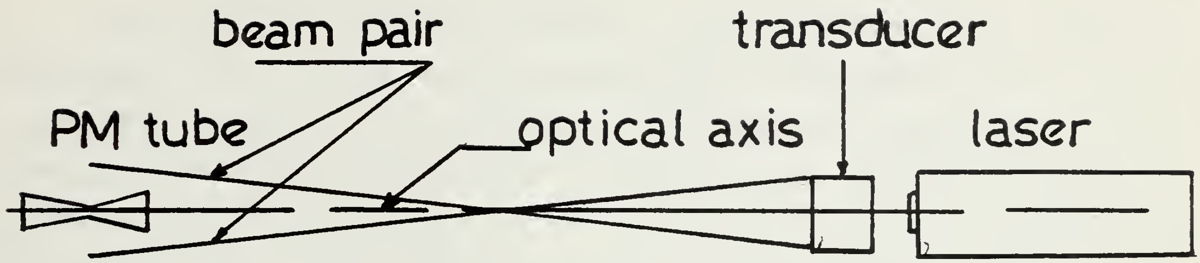


V_∞
$t_1 = \frac{5}{8} t_2$
t
t_1
t_2
validated signal
t
t_1
t_2
non-validated signal

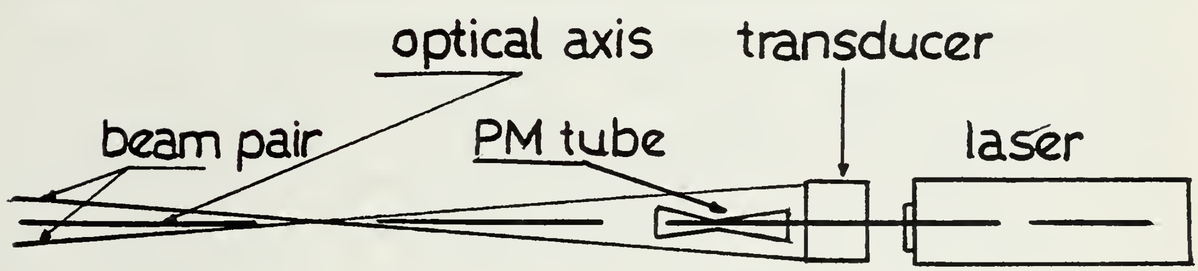
Figure 2

signal.

Back scatter mode: again the photomultiplier tube is mounted on the optical axis, but now it is aligned in the direction of the emerging beam pairs. This mode affords the ability tc measure fluid velocities in confined spaces or in situations where the dectector cannot be mounted in fcrward scatter, such as in measuring velocities in a compressor. Additionally, operation in this mode requires a higher powered laser due to the reduced intensity of back scattered light (Fig. 3).



LDV FORWARD SCATTER MODE



LDV BACK SCATTER MODE

Figure 3

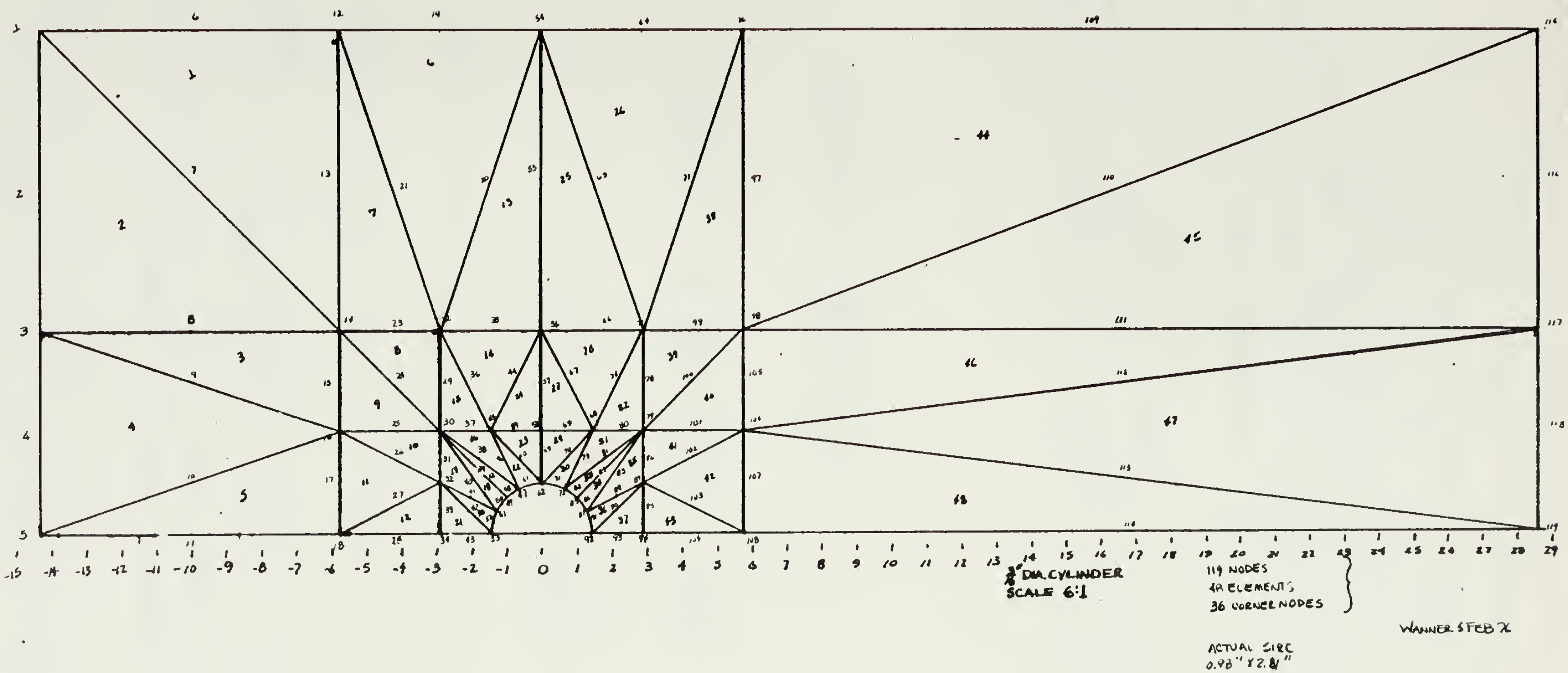
III ANALYTICAL DERIVATION

The finite element method is a numerical analysis technique fcr obtaining approximate solutions to a wide variety of enginering problems. It offers a means to solve complex continuum problems by sub-dividing them into simpler interrelated problems. The nature of this engineering problem does not readily lend itself to a closed form solution. Therefore a numerical solútion is dictated.

The method of finite elements is well employed where there are irregular geometries or unusual specifications of boundary conditions. The method gives a piecewice approximation to the governing equations. Figure 4 shows the finite element grid employed in this flow problem. Approximating functions (interpolation functions) are defined in terms of the values of the field variables at the nodes. For the finite element representation, the nodal values of the field variables become the new unknowns and, once found, the interpolation functions define the field variable throughout the field of elements. The degree of the interpolaticn polynomials is governed by the number of nodes per element and dictated by the order of the governing equations.

The basic principle of the finite element method is to divide the solution domain into a finite number of subdomains (elements) which are connected only at node points. Interpolation polynomials are created for each element and a collecton of interpolation functions over the whole soluticn domain provide a piecewise approximation to the field variable.

Introduction of generalizations facilitate direct derivation of finite element equations from governing differential equations. The method of weighted residuals



Finite element grid

Figure 4

(Galerkin's Method) is a technique for obtaining approximate solutions to non-linear partial differential equations. We first assume the general function behavior of the dependent field variable in some way so as to approximately satisfy the given differential equations and boundary conditions. After applying this approximation to the original equations, some error will result, which we desire to minimize or drive to zero over the entire solution domain.

The basic set of differential equations governing the problem from a velocity and pressure formulation approach are (after linearization by approximating non-linear viscous terms u_n, v_n):

continuity $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

momentum $$u_n \frac{\partial u}{\partial x} + v_n \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu \nabla^2 u$$

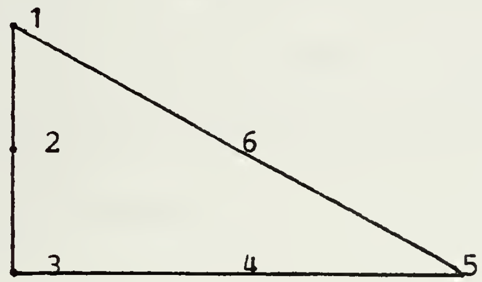
$$u_n \frac{\partial v}{\partial x} + v_n \frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu \nabla^2 v$$

u_n, v_n, and P_n are approximate solutions to the flow problem and may be visualized as the n^{th} iteration of the procedure of solution. These equations used in formulation of the problem are by no means unique. Stream functions and stream function with vorticity formulations have also been employed to establish finite element solution procedures.

Inasmuch as the velocity terms are of second order, a second order interpolation polynomial must be used to

calculate these unknowns. The pressure terms are of first order and hence require only first order interpolation.

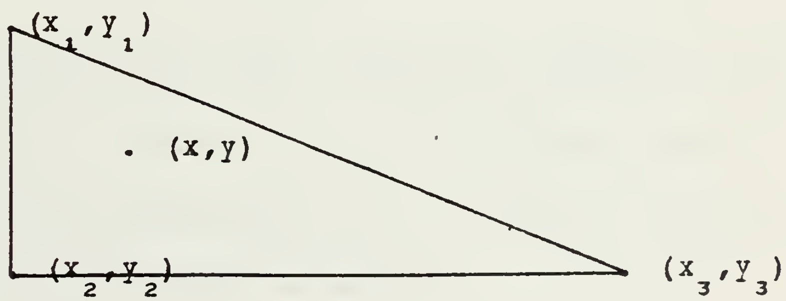
The basic element chosen for this solution was the triangle and, in a localized scheme, is visualized as follows:



Nodes 2, 4 and 6 lie equidistant between their respective corner nodes.

Nodes 1, 3, and 5 are referred to as corner nodes and are the basis for the linear interpolation polynomials derived for the pressure terms. For any triangular element, the corner nodes are described by their Cartesian coordinates x_i, y_i.

Consider the following local element:



The coordinates of node one are x_1, y_1; node two x_2, y_2; etc. and the interior point (x,y) may be described as a linear combination of the three corner nodes:

$$x=L_1x_1+L_2x_2+L_3x_3$$

$$y=L_1y_1+L_2y_2+L_3y_3$$

where L_i is the natural coordinate (interpolation polynomial, here linear) or weighting function which relates the coordinates of the corner nodes to any interior node. L_1 has a value of one at the point x_1,y_1 and is zero at the other two nodes of the element.

The L_i's expressed as linear functions of the Cartesian coordinates then are

$$L_1(x,y)= \frac{1}{2\Delta}(a_1+b_1x+c_1y)$$

$$L_2(x,y)= \frac{1}{2\Delta}(a_2+b_2x+c_2y)$$

$$L_3(x,y)= \frac{1}{2\Delta}(a_3+b_3x+c_3y)$$

$$\text{where } 2\Delta = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

which is equal to the area of the triangular element and $a_1=x_2y_3-x_3y_2$, $b_1=y_2-y_3$, and $c_1=y_1-y_2$, the determinants of the minors corresponding coefficients a_1, b_1, or c_1, and the other coefficients are similarly computed.

For the velocity terms, second order interpolation polynomials are indicated, while the pressure terms remain first order. Hence the introduction of more nodes (6) will result in a new set of weighting functions which may also be

expressed in terms of the natural coordinate interpolation polynomials:

$$N_1 = 2L_1^2 - L_1 \qquad N_2 = 4L_1 L_2$$

$$N_3 = 2L_2^2 - L_2 \qquad N_4 = 4L_2 L_3$$

$$N_5 = 2L_3^2 - L_3 \qquad N_6 = 4L_1 L_3$$

$$N_i^P = L_i$$

$$u^e = \sum_1^6 N_i u_i \qquad v^e = \sum_1^6 N_i v_i \qquad P^e = \sum_1^3 N^P P_i \quad \text{(xxxx)}$$

Application of Galerkin's Method to the governing equations results in the following intergral expressions:

$$-\int_\Omega N_i \left(u_n \frac{\partial u}{\partial x} + v_n \frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial x} - \nu \nabla^2 u\right) d\Omega = 0$$

$$-\int_\Omega N_i \left(u_n \frac{\partial v}{\partial x} + v_n \frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial y} - \nu \nabla^2 v\right) d\Omega = 0$$

$$\int_\Omega N_i^{(P)} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) d\Omega = 0$$

here Ω implies "over the area of the triangle".

Then we may make the substitution of the field variable in terms of x and y into terms of L_i.

e.g.: given $u(x,y) \Leftrightarrow u(L_i)$

then $\dfrac{\partial u}{\partial x} = u^{(e)} \dfrac{\partial N_i}{\partial L_i} \dfrac{\partial L_i}{\partial x}$

where $\dfrac{\partial L_i}{\partial x} = \dfrac{b_i}{2\Delta}$

and a similar procedure for $\dfrac{\partial u}{\partial y} = u^{(e)} \dfrac{\partial N_i}{\partial L_i} \dfrac{\partial L_i}{\partial y}$

where $\frac{\partial L_i}{\partial y} = \frac{c_i}{2\Delta}$

Substitution of equations (xxxx) into u^e, v^e and P^e and application of the above chain rule allows integration by parts:

$$K_1 = \int \left[\frac{\mu}{\rho} \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) - u^{(e)} N_i \frac{\partial N_j}{\partial x} - v^{(e)} N_i \frac{\partial N_j}{\partial y} \right] dx\,dy$$

$$K_2 = -\int N_j^{(P)} \frac{\partial N_i}{\partial x}\, dx\,dy$$

$$K_3 = -\int N_j^{(P)} \frac{\partial N_i}{\partial y}\, dx\,dy$$

The evaluated integral expressions then form the influence matrix as follows:

$$\left[\begin{array}{c|c|c} K_1 & 0 & K_2 \\ \hline 0 & K_1 & K_3 \\ \hline K_2 & K_3 & 0 \end{array}\right]$$

Specification of the field variables or right hand sides of the matrix equation then provides a defined problem:

$$[TM] \cdot [T] = [Q]$$

Additionally, evaluation of the integral expressions yields boundary terms which must be computed for nodes which lie on the boundary of the control surface:

$$R_1 = \int N_i X^* ds \qquad R_2 = \int N_i Y^* ds$$

where $X^* = \mu \nabla u^{(e)} \cdot \hat{n} - \hat{n}_x P^{(e)}$ and $Y^* = \mu \nabla v^{(e)} \cdot \hat{n} - \hat{n}_y P^{(e)}$

It should be noted that along vertical boundaries there is

no outward pointing normal $\hat{n}_y$ and similarly along horizontal boundaries no $\hat{n}_x$ exists, and furthermore these terms are not computed where u and v are specified. Additionally, it was assumed that along the left-and right-hand side and upper boundaries $\partial u/\partial x$ and $\partial v/\partial y$ were identically equal to zero because these boundaries were placed at five or more characteristic lengths (diameters) from the cylinder and considered to be outside the realm of influence of the flow about the cylinder.

The terms comprising K_1, K_2 and K_3 are entered element by element into the main program in steps FLU02080 through QZ00170. Then each element's matrix is compiled into the field matrix (comprised of all elemental matricies) and is solved iteratively to output the horizontal velocity component u, the vertical component v, and the pressure P at every node in the field.

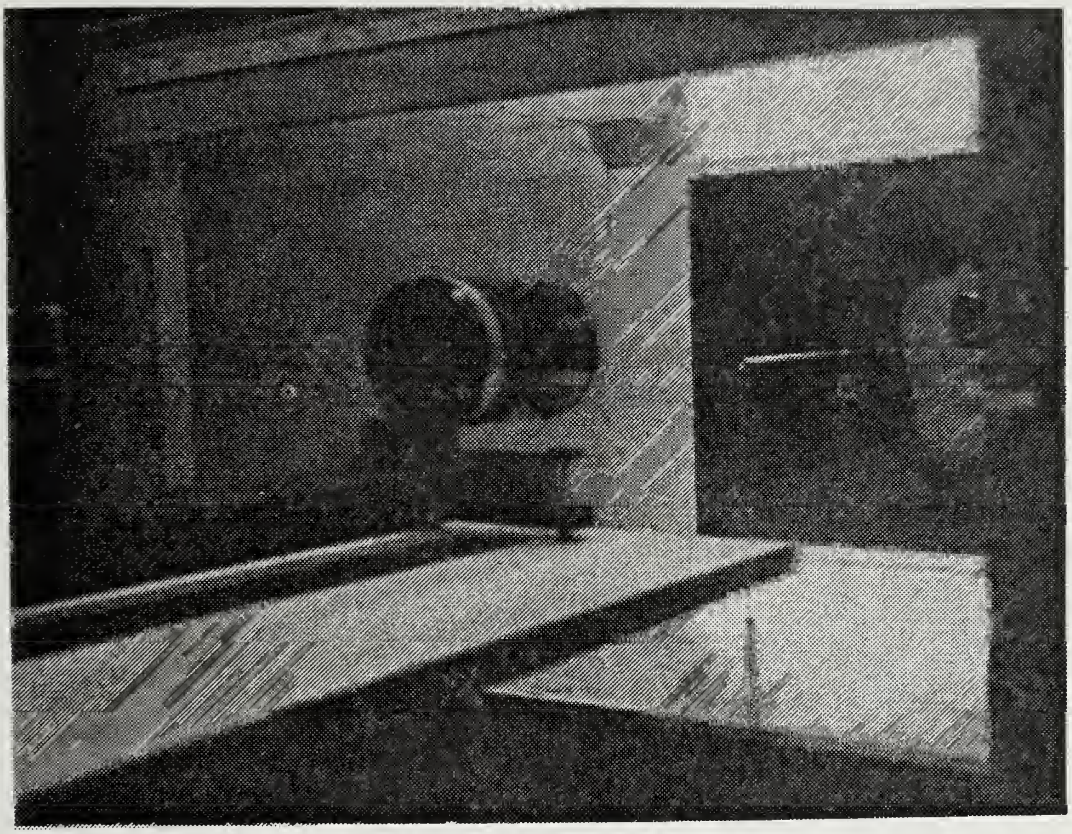
IV. DESCRIPTION OF EXPERIMENT

A right-angled, three-sixteenths inch cylinder made of stainless steel was mounted in an AEROLAB series 90 32x45 inch low speed wind tunnel through the floor of the test section (Fig. 5). The portion of the rod protruding down through the test section was mounted on a two-degree-of-freedom, micrometer controlled bed which allowed the rod to be maneuvered in the vertical plane parallel to the flow direction. This scheme enabled the laser velocimeter and photomultiplier tube to be mounted, aligned and left undisturbed for the entire period during which data were taken. The bed was positioned to read locations in the control volume which directly coincided with the location of the nodes in the finite element mesh (Fig. 6).

In the interest of precluding as many opportunities for error as possible, all u velocity components were first read and then components in a forty-five degree position were taken thus necessitating only one re-orientation of the beam pairs.

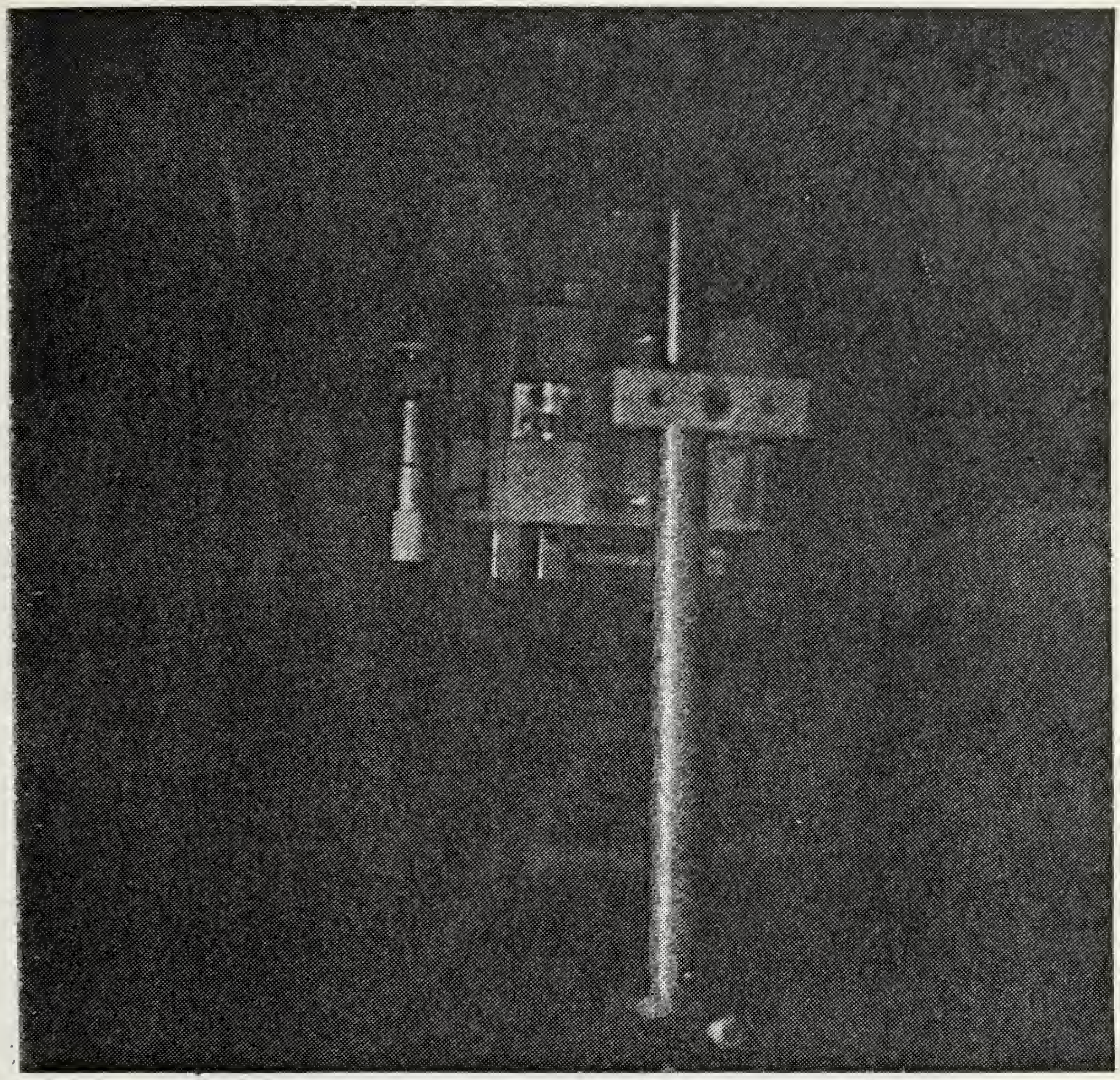
The plexiglass window prefacing the optics transducer of the test section had one-quarter inch holes drilled into it to allow the beam pairs to pass unimpeded, thereby minimizing distortion of the focal volume fringe pattern. The photomultiplier tube was likewise accommodated through the far window.

The seeding mechanism employed zinc-hexaclorcthane based (non-toxic) white smoke bombs. The nature of the closed return wind tunnel permitted continuous measurement without interruption for re-seeding of the flow. By means of electron microscopy, the average size of the smoke particles was determined to be on the order of two microns (Fig. 7).



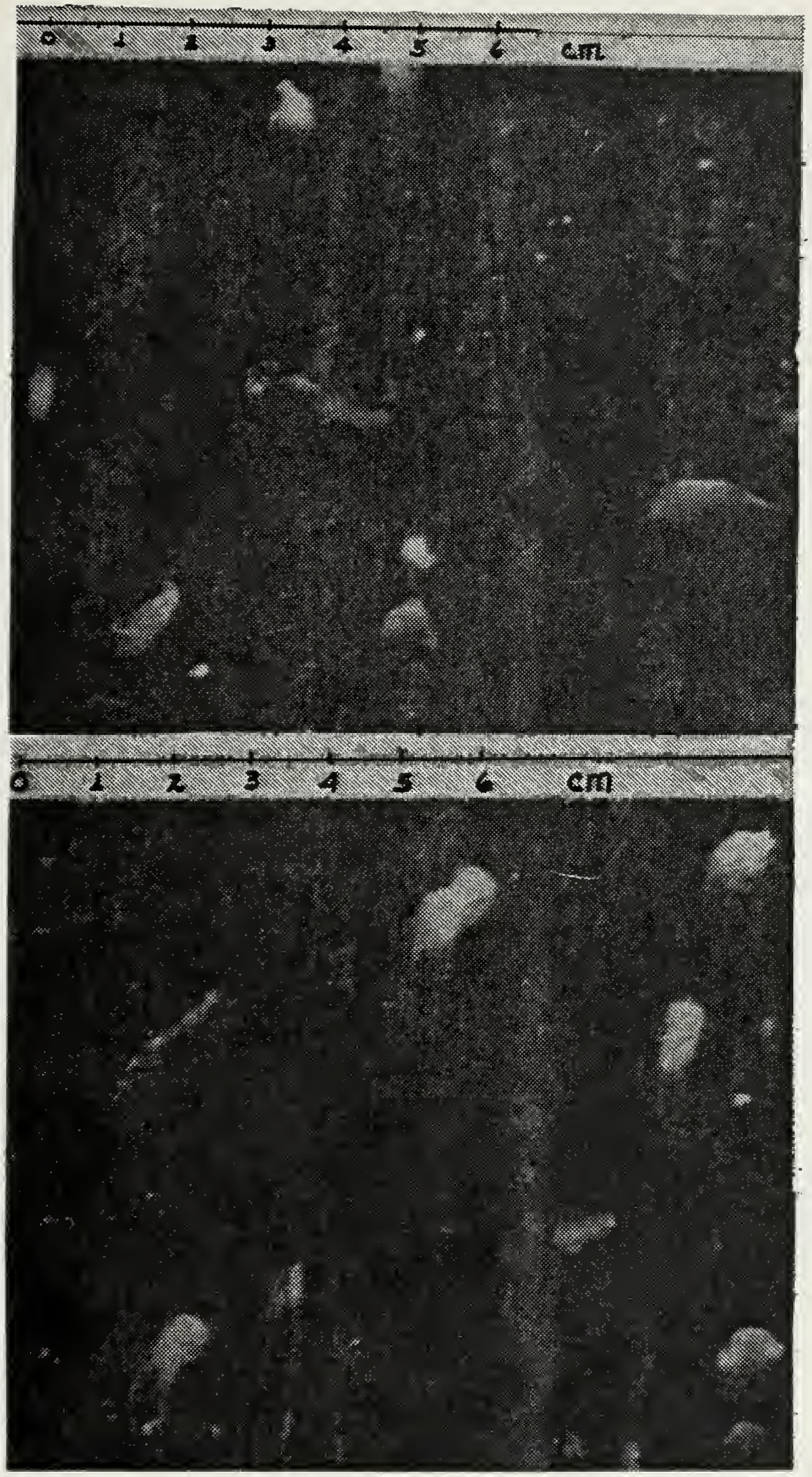
Cylinder mounted in wind tunnel and LDV setup.

Figure 5



Traversing mechanism

Figure 6

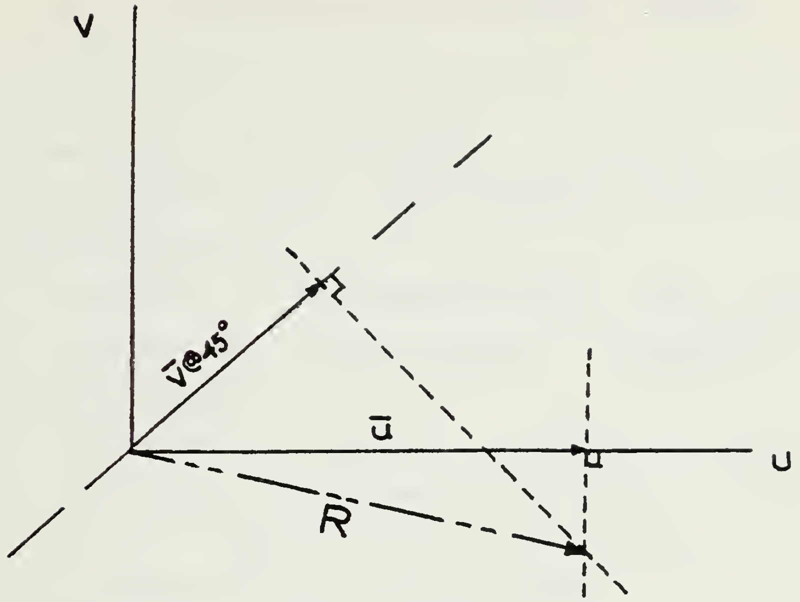


Zinc-hexachlorathane particles
(X5000 1cm= 2.0 μm)

Figure 7

The final velocity component measured at each node was the result of averaging three sucessive readings from the velocimeter processor. Each of these readings represented the average of 4096 readings which had been validated to within 1.5% of the exact five-eighths validation count criteria.

The flow in the wind tunnel was stabilized at five inches per second to maintain a Reynolds number of about 40, which is in the realm of creeping flow. The experimental setup was designed to measure the steady flow about the cylinder, hence the relatively low Reynolds number, and no effort was made to obtain unsteady flow measurements. Three separate runs were conducted, two over the forward half of the control volume and one over the rear half. Because the beams of the laser anemometer were approximately one-sixteenth inch in diameter, several node position measurements were excluded due to inaccessibility. The majority of these were in close proximity to the cylinder. In order to obtain both an x- and a y-velocity component of the flow at every node which was accessible, two measurements were made, but because of the very small vertical component present in most of the flow, direct vertical beam pair measurement was unsuitable. Instead, after obtaining the horizontal component for each node in a run, a second series of measurements was made with the beam pairs inclined forty-five degrees from the vertical. In this fashion the y-component was determined by the following relationship:



It can be seen here that the unknown resultant (R) has a projection onto each axis representing the orientation of the beam pairs, thus the resultant is the vector which emanates from the origin and terminates at the intersection of the perpendiculars constructed from the $\bar{u}$ and $\bar{v}$(at 45 degrees) vectors.

V. DESCRIPTION OF COMPUTER PROGRAM

The program found in the appendix has several comment cards inserted for clarification. However, a broader presentation is offered here. The program may be subdivided into four main sections:

Section 1: dimensioning and zeroing of parameters and arrays. (FLU00020 through FLU00690)

Section 2: input of node coordinates, system topology and known values of u, v, P, and right hand sides, and output of the same. (FLU00700 through FLU02090)

Section 3: computation of interpolation polynomials on an elemental level and arrangement into local element arrays. (FLU02100 through G000320)

Section 4: loading of the global influence matrix (TM) and solution of (TM)(T)=(Q) by Gauss-Jordan partial pivoting and back substitution, with an iterative routine incorporated until convergence is attained, and associated print out. (FLU04470 through FLU04880)

The computer routine systematically tested the solution of field variables with the previous solution until all terms in the T and T1 arrays agreed within 0.001 (ESPILA). If the convergence test failed, the process was repeated from section 3.

Initially, the program was run on an IBM 360 in FORTRAN CLG mode, then an ensuing run was completed on the FORTRAN HCLG compiler and a time reduction of two-thirds was realized. The program, as given in the appendix, required approximately 375K bytes of core to compile and 675K bytes

to execute and output. Substantial space savings may be realized by utilizing banded matrix storage methods vice the method employed here, but due to time limitations this avenue was not fully explored.

The program attempts to solve simultaneously for all the field variables. However, introduction of zeros on the main diagonal of the TM influence matrix resulted in a singular matrix. Another approach was explored where the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

was replaced by

$$-\frac{1}{2}\nabla^2 P = \left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} \right]$$

and the resulting terms were added to the TM matrix and their respective right hand sides. The only field variables affected by this form of the continuity equation were the pressure variables, and depending on whether the local corner node involved in the local pressure equation was on the boundary or interior, different right hand sides had to be evaluated. This is the purpose of steps FLU04460 through GO00320. The simultaneous solution technique for all field variables applied to this new set of equations did not converge.

A third, unimplemented technique may be considered. Given the original matrix equations:

$$(1) \quad K_1 U + 0 + K_2 P = R_1$$

$$(2) \quad 0 + K_1 V + K_3 P = R_2$$

$$(3) \quad K_2 U + K_3 V + 0 = 0$$

Solution for u and v will result in

$$(1) \quad U = -K_1^{-1} K_2 P + K_1^{-1} R_1$$

$$(2) \qquad V = -K_1^{-1} K_3 P + K_1^{-1} R_2$$

and multiplication by K_2 and K_3 yields

$$K_2 U = -K_2 K_1^{-1} K_2 P + K_2 K_1^{-1} R_1$$

$$K_3 V = -K_3 K_1^{-1} K_3 P + K_3 K_1^{-1} R_2$$

Summation of these equations allows substitution for the third matrix equation:

$$\left[K_2 K_1^{-1} K_2 + K_3 K_1^{-1} K_3 \right] P = K_2 K_1^{-1} R_1 + K_3 K_1^{-1} R_2$$

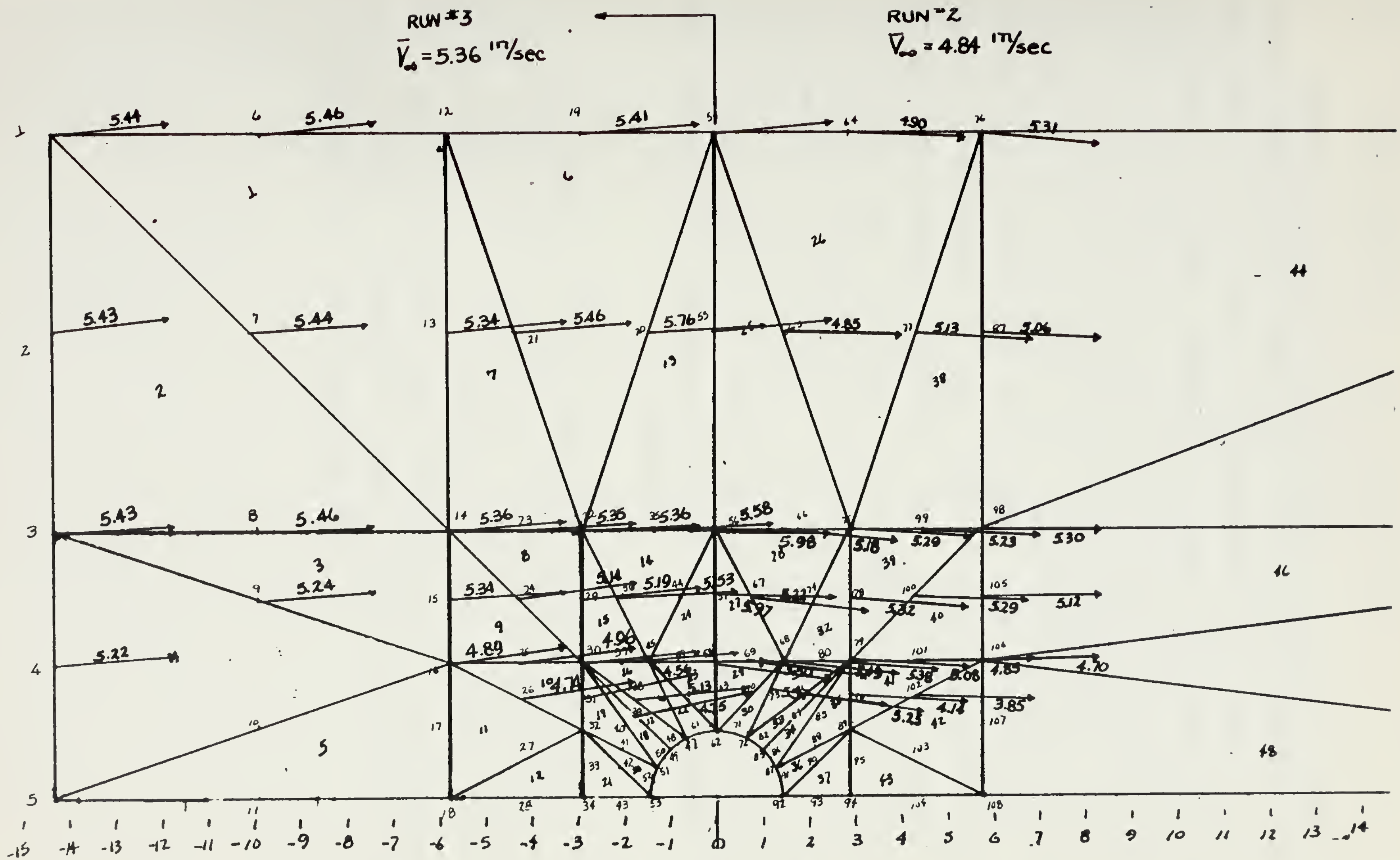
The matrix operating on P may be calculated on an algebraic level and entered, in terms of a_i, b_i and c_i, into the program in order to solve for the pressures at every corner node. The calculated P's then would be entered into the field variable array and the partial pivoting routine, which involves only those terms where the field variable is not specified, would operate primarily on the first two matrix equations. In previous computer trials where all corner node pressures were specified, although not known to be correct, calculations were properly performed. This method would allow the actual pressures present in the control surface to be entered into the computer routine for further calculation of the velocity components.

VI. ANALYSIS OF RESULTS

No computer routine output was obtained for comparison of results, due to the inability of the program to converge. As such, comparison of the two methods cannot be offered here. The non-convergence of the program was investigated from several aspects: validity of the non-linear terms, correctness of the loading of the TM matrix on both the elemental and global levels, correctness of input, performance of the partial pivoting subroutines and the iteration scheme; all without success. In test cases the program initially solved for the unknowns within magnitude accuracy for a majority of terms. However, sucessive iterations resulted in oscillating solutions, even after twenty iterations. Convergence was expected after ten to fifteen iterations without under- or overrelaxation factors incorporated into the iteration scheme. After the new formulation of the continuity equation was incorporated into the computer routine, the program execution again resulted in a singular matrix when only the left and right hand and upper control surface boundaries had their respective pressures specified. Underflow occurred in the execution of the subroutine SOLVE at step SOL00260. Subsequent investigation revealed that the 178^{th} and 179^{th} rows of the reduced matrix equations were the cause of the underflow.

The data which were taken for runs number two and three are displayed in the appendix and are plotted on the finite element grid in Figure 8. Without the analytical results to compare the measured data only a qualitative statement may be made pertaining to the LDV output. It is readily seen that the trend of flow directions around the cylinder appears to be correct. By and large, the flow in close proximity to the cylinder was accelerated over the top and decelerated near the stagnation points. It is pointed out

here that the plot is a composite of two different trials, and as such, different free stream velocities were present for each run. In Figure 8 the vector representing the flow at a node was plotted and the magnitude of the vector in inches per second was displayed above the vector or by the node from which the velocity vector emanated.



Velocity data plot

Figure 8

A. ERROR ANALYSIS

Re-positioning of the cylinder to its base location was accomplished with relatively low position error. Prior to each run the cylinder was aligned using the beam pairs as reference. In the horizontal mode, the beam pairs would strike the leading and trailing portions of the cylinder and reflect to positions horizontally opposed on the far side of the wind tunnel when correctly zeroed vertically. The same procedure was repeated for the vertical beam pair except position was checked by the upper beam reflecting to a vertical position on the far wall and the lower beam (at reduced intensity) striking the cylinder support on centerline. This procedure was repeated several times and the maximum deviation between any aligned positions was 0.2 millimeters horizontally or vertically. Thus, it was determined that the maximum positioning error inherent in the system would be the ratio of the smallest coordinate of any node measured and the aforementioned 0.2 millimeters or

$$0.2/3.75=0.5\%$$

Freestream velocity measurements were likewise tested for fluctuations. Typical measurement variations ranged from 0.129 to 0.142 (run number three; freestream measurements) in the horizontal mode and 0.0947 to 0.100 in the forty- five degree mode (velocities in meters/seconds). These figures represent a total of 10% variation which may be considered as ± 5.% from the mean.

VII. CONCLUSION

The experiment, as described, required approximately two to four hours for each run. aside from the initial alignment of the components and the cylinder, this was perhaps the greatest drawback of the system. Next, the exclusion of regions in close proximity to the cylinder from measurement was deemed to be a major factor in the small amount of data actually taken. The latter may be improved with a different experimental arrangement; particularly, the plexiglass section directly prefacing the optics transducer should be completely removed, thus allowing all angles of beam pair inclination, and in turn availing more locations to measurement. Another improvement which could be implemented would be unitization of the laser, transducer and photomultiplier tube arrangement. This would decrease the LDV's sensitivity to vibration and provide a more accurate optical axis alignment. The aforementioned improvements would still not decrease the time required for data collection, but the introduction of a two component system [Ref.4] would allow simultaneous measurement using orthogonal beam pairs. Furthermore, this type of system would not concern itself with the errors introduced in beam pair re-alignments performed with the single beam pair LDV.

The reproducibility of the experiment was judged to be good with the exception of exact duplication of the wind tunnel velocity between successive trials. This, of course, is primarily a function of the controllability of the tunnel motor and propellor assembly. It is estimated that with adequate facilities for incorporation of some of the aforementioned improvements, more accurate data may be obtained. The LDV displays great potential as a means of measuring a wide variety of dynamic fluid regimes accurately and without introduction of disturbance to the flow itself.

APPENDIX A

LDV DATA

RUN #2

Node	U(cm/sec)	V@45°(cm/sec)	R(in/sec)	∠°
64	12.43	8.43	4.90	-2.3
65	12.33	8.49	4.85	-1.5
66	13.10	8.33	5.18	-5.8
67	13.17	8.25	5.22	-6.5
68	13.87	8.72	5.49	-6.3
69	13.87	8.56	5.50	-7.2
73	14.03	8.58	5.57	-7.7
75	13.40	8.83	5.29	-3.9
76	13.43	8.62	5.31	-5.3
77	13.00	8.62	5.13	-3.6
78	13.47	8.87	5.32	-4.5
79	12.87	8.58	5.08	-3.3
80	13.63	8.91	5.38	-4.3
81	13.20	8.02	5.25	-8.0
96	10.50	6.94	4.14	-3.7
97	12.83	8.64	5.06	-2.7
98	13.47	9.54	5.30	0.1
99	13.27	8.92	5.23	-2.8
100	13.43	9.25	5.29	-1.5
101	12.33	8.83	4.85	0.7
102	9.77	6.69	3.85	-1.8
105	13.03	9.32	5.12	0.8
106	11.93	8.68	4.70	1.7

RUN #3

Node	U(cm/sec)	V@45°(cm/sec)	R(in/sec)	°
1	13.73	10.77	5.44	6.23
2	13.70	10.43	5.43	6.25
3	13.77	10.33	5.43	3.49
4	13.23	10.03	5.22	4.11
6	13.77	10.93	5.46	7.00
7	13.80	10.40	5.44	3.77
8	13.87	10.17	5.46	2.11
9	13.27	10.08	5.24	4.27
13	13.50	10.47	5.34	5.54
14	13.57	10.40	5.36	4.80
15	13.53	10.20	5.34	3.76
16	12.30	9.93	4.89	8.05
19	13.70	10.43	5.41	4.38
20	14.60	10.87	5.76	3.02
21	13.83	10.57	5.46	4.63
22	13.60	10.06	5.36	2.65
23	13.57	10.23	5.35	3.79
24	12.93	10.40	5.14	7.84
25	12.50	9.92	4.96	7.00
26	11.90	9.64	4.74	8.27
29	13.13	10.16	5.19	5.40
30	12.57	9.80	4.98	5.86
31	11.30	9.76	4.56	12.50
35	14.17	10.47	5.58	2.60
36	14.03	10.43	5.53	4.36
37	13.23	10.07	5.22	4.36
38	12.97	10.09	5.13	5.72
39	11.83	10.03	4.75	11.24
54	14.03	10.87	5.55	5.58
55	14.17	11.03	5.61	5.76
56	15.20	10.57	5.98	-0.90
57	15.17	10.47	5.97	-1.36
58	16.27	10.00	6.46	-7.46
59	14.37	10.20	5.66	0.20

```
C
C
C
C     THIS RUN HAS KINEMATIC VISCOSITY =.022948 (INCHES**2)
C      THIS RUN (#2) HAS A REYNOLDS NO.= 39.59
C
         DATA NREAD/5/
C                                                                       FLU00020
C     THE U VELOCITY IS IN THE FIRST NN POSITIONS OF  T(I) (NN=NUMBER OFNODFLU00030
C     THE V VELOCITY IS IN THE SECOND NN POSITIONS OF T I.E. T(I+NN)    FLU00040
C     THE P PRESSURE IS IN THE  NN+NN+I POSITIONS OF  T  I.E.  T(I+NN+NN)  FLU00050
C       THERE ARE  NNCN  PRESSURE NODES (NNCN=NUMBER OF CORNER  NODES)  FLU00060
C                                                                       FLU00070
C      BOTH TM AND B MUST TO DIMENSIONED 2*NN+NNCN X 2*NN+NNCN          FLU00080
C                                                                       FLU00090
C                                                                       FLU00100
            DATA NWRITE/6/                                              FLU00110
         DATA STOP/'STOP'/                                              FLU00120
C       PRESSURE READ IN FOR RUN # 2
         DATA PRES/14.8/
         DIMENSION TM(274,274),B(274,274)
         DIMENSION XC(120),YC(120),NODE(120,6),NVS(120),NCN(120)
         DIMENSION T(274),NVIS(274),NCP(120),NPS(120),Q(274)
         DIMENSION NQS(120),T1(274)
         DIMENSION TM$(15,15),N(15),NQIS(274)
         DIMENSION Z(274),C(274)
         DIMENSION RHS(274)
         DIMENSION RP$(6),ZP$(6)                                        FLU00200
         DIMENSION XC$(3),YC$(3)                                        FLU00210
           DIMENSION U(6),V(6),QZ1(6,6),QZ2(6,6),QZ3(6,6)               QZ 00020
          DIMENSION NI(18),NT(3)
         DIMENSION NLCY(4),NRCY(4),QZB(6),QZC(6)                        GO 00010
C                                                                       FLU00220
C      IF NCASE EQUALS 1, THEN THE PROBLEM WILL INCLUDE
C        NCN-LINEAR TERMS AND A TWO DIMENSIONAL SOLUTION
C(                                                                      FLU00250
         READ(NREAD,500)NCASE                                           FLU00260
         IF(NCASE.EQ.1)GO TO 5                                          FLU00270
         GO TO 6                                                        FLU00290
       5 WRITE(NWRITE,2020)                                             FLU00300
       6 CONTINUE                                                       FLU00310
C                                                                       FLU00320
C     THE FIRST PART OF THE PROGRAM CAN BE CONSIDERED AS AN INPUT ROUTINE  FLU00330
C     LINE 022 TO  0170 ARE INPUT AND VERIFICATION OF ALL DATA          FLU00340
C                                                                       FLU00350
C     THIS WOULD BE PART OF ANY FINITE ELEMENT PROGRAM                  FLU00360
C                                                                       FLU00370
C                                                                       FLU00380
```

```
C     READ IN NUMBER OF NODES AND ELEMENTS AND NO. CORNER NODES         FLU00390
         READ(NREAD,1005)NN,NE,NNCN                                     FLU00400
C                                                                       FLU00410
C        INITIALIZE ALL PARAMETERS                                      FLU00420
C                                                                       FLU00430
           MM=NN+NN+NNCN                                                FLU00440
         DC 50 I=1,120
         XC(I)=0.                                                       FIX00020
         YC(I)=0.                                                       FIX00030
         NVS(I)=0                                                       FIX00040
         NCN(I)=0                                                       FIX00050
         NCP(I)=0                                                       FIX00070
         NPS(I)=0                                                       FIX00080
         NCS(I)=0                                                       FIX00200
   50    CONTINUE                                                       FIX00090
         DO 51 I=1,MM                                                   FIX00100
         NVIS(I)=0                                                      FIX00060
         NQIS(I)=0                                                      FIX00210
         T(I)=0.                                                        FIX00110
         Q(I)=0.                                                        FIX00120
         T1(I)=0.                                                       FIX00130
         Z(I)=0.                                                        FIX00140
         C(I)=0.                                                        FIX00150
         DO 51 J=1,MM                                                   FIX00160
         B(I,J)=0.                                                      FIX00170
   51    CONTINUE                                                       FIX00180
         DO 53 I=1,15                                                   FIX00230
         N(I)=0                                                         FIX00240
         DO 53 J=1,15                                                   FIX00250
         TM$(I,J)=0.                                                    FIX00260
   53    CONTINUE                                                       FIX00270
         DO 54 I=1,6
         RP$(I)=0.                                                      FIX00290
         ZP$(I)=0.                                                      FIX00300
         U(I)=0.0
         V(I)=0.0
         DO 54 K=1,6
         QZ1(I,K)=0.0
         QZB(I)=0.0
         QZC(I)=0.0
         QZ2(I,K)=0.0
         QZ3(I,K)=0.0
   54    CONTINUE                                                       FIX00310
         DO 55 I=1,3                                                    FIX00320
         XC$(I)=0.                                                      FIX00330
         YC$(I)=0.                                                      FIX00340
         NT(I)=0
   55    CONTINUE                                                       FIX00350
```

```
      DO 56 I=1,4
      NLCY(I)=0.0
      NRCY(I)=0.0
   56 CONTINUE
C  READ NODE NUMBERS AND COORDINATES
C
      DO 100 J=1,NN
      READ(NREAD,1006)WORD,I,XC(I),YC(I)
       IF (WORD.EQ.STOP) GO TO 101
        NCN(J)=I
  100 CONTINUE
  101   NNCN=J-1
C
C  THE ARRAY  NCP(J) GENERATES THE GLOBAL PRESSURE INDICES (P1,P2.)
C  THUS PRESSURE NODES ARE LABELED AS CORNER NODES ARE INPUTTED
C     WHEN ONE INPUTS A GLOBAL CORNER NODE FOR J
C
      DO 107 J=1,NNCN
      NCP(NCN(J))         =J+NN+NN
  107 CONTINUE
C
C   SYSTEM TOPOLOGY( ELEMENT NO. AND NODE NUMBERS IN
C COUNTER-CLOCKWISE FASHION STARTING AT ANY CORNER NODE
C ALWAYS COUNT FROM UPPER LEFT HAND CORNER
C
      DO 105 I=1,NE
      READ(NREAD,1010)J,NODE(J,1),NODE(J,2),NODE(J,3),
     1NODE(J,4),NODE(J,5),NODE(J,6)
  105 CONTINUE
         MAXDIF=0
      DO 108 I=1,NE
      DO 108 J=1,6
      DO 108 K=1,6
         LL=IABS(NODE(I,J)-NODE(I,K))
         IF(LL.GT.MAXDIF) MAXDIF=LL
      IBAND=2*(MAXDIF+1)
      NEQ=2*NN+NNCN
 108  CONTINUE
      WRITE(NWRITE,1017)IBAND,NEQ
 1017 FORMAT(5X,'IBAND=',I3,//,5X,'NEQ =',I3,//)
C
C  READ NODES WHERE BOTH U AND V VELOCITY IS SPECIFIED
C
      DO 110 I=1,MM
      READ(NREAD,1006)WORD,NVELS,VELU,VELV
       IF(WORD.EQ.STOP) GO TO 111
      NVS(I)=NVELS
      T(NVS(I))=VELU
```

```
      T(NVS(I)+NN)= VELV
 110     CONTINUE
C
C  COUNT NODES HAVING SPECIFIED  VELOCITIES
C
  111 NNVELS= I-1
C
C
C
C READ QX AND QY VALUES AT INTERNAL NODES
      DO 125 I=1,NN
        READ(NREAD,1006)WORD,NQXY,QXNS,QYNS
        IF(WORD.EQ.STOP) GO TO 126
        NQS(I)= NQXY
        Q(NQS(I))=QXNS*PRES
        Q(NQS(I)+NN)=QYNS*PRES
  125 CONTINUE
  126   NNQXY=I-1
C
C  READ NODE NUMBERS AND  PRESSURE WHEREE PRESSURE  IS SPECIFIED
C
      DO 130 I=1,NN
      READ(NREAD,1025)WORD,NP,PNP
      IF(WORD.EQ.STOP)GO TO 135
      NPS(I)= NP
      T(NCP(NPS(I)))= PNP
  130 CONTINUE
C
C COUNT  BOUNDARY NODES WHERE PRESSURE SPECIFIED
C
  135 NNPS= I-1
C
C
C READ PRESSURE NODES WHERE Q IS SPECIFIED
C
C
      DO 141 I=1,MM
      READ(NREAD,1025)WORD,NPQ,QNPQ
      IF(WORD.EQ.STOP)GO TO 142
      NPS(I+NNPS)=NPQ
      Q(NCP(NPS(I+NNPS)))=QNPQ
  141 CONTINUE
  142 NNPQ=I-1
C
C    READ IN CORNER NODES WHICH ARE CONTAINED IN THE INTERIOR
C
        DO 143 I=1,MM
        READ(NREAD,7654) NINT
```

```
      IF(WORD.EQ.STOP) GO TO 147
      NI(I)=NINT
  143 CONTINUE
 7654 FORMAT(6X,A4,14I4)
  147 CONTINUE
C
C NCIS IS A LIST OF THE INDICES OF KNOWN QX QY
C
      DO 1140 I=1,NNQXY
      NQIS(I)=NQS(I)
       NQIS(I+NNQXY)=NQS(I)+NN
 1140  CONTINUE
     DO 1142 I=1,NNPQ
     NQIS(I+2*NNQXY)=NPS(I+NNPS)+2*NN
 1142 CONTINUE
C
C  NVIS IS A LIST OF INDICIES OF KNOWN VELOCITIES AND PRESSURES
C
      DO 1150 I=1,NNVELS
      NVIS(I)=NVS(I)
      NVIS(I+NNVELS)=NVS(I)+NN
 1150 CONTINUE
      DO 1160 J=1,NNPS
     NVIS(2*NNVELS+J)=NCP(NPS(J))
 1160   CONTINUE
C
C NTOTQ TOTAL NUMBER OF KNOWN QX AND QY
C
      NTOTQ=2*NNQXY +NNPQ
      NTOTVP=2*NNVELS+NNPS
C
C  PRINT ALL INPUT DATA
C
      WRITE(NWRITE,1035)NN,NE,NNCN
      WRITE(NWRITE,1040)
      DO 150 I=1,NNCN
       WRITE(NWRITE,1045)NCN(I),XC(NCN(I)),YC(NCN(I))
  150   CONTINUE
      WRITE(NWRITE,1050)
      DO 155 I=1,NE
      WRITE(NWRITE,1055)I,NODE(I,1),NODE(I,2),NODE(I,3),
     1NODE(I,4),NODE(I,5),NODE(I,6)
  155 CONTINUE
      WRITE(NWRITE,1060)
      DO 160 I=1,NNVELS
      WRITE(NWRITE,1065)I,NVS(I),T(NVS(I)),T(NVS(I)+NN)
  160 CONTINUE
      WRITE(NWRITE,1070)
```

```
      DO 165 I=1,NNQXY                                                  FLU01890
       WRITE(NWRITE,1065)I,NQS(I),Q(NQS(I)),Q(NQS(I)+NN)                FLU01900
  165 CONTINUE                                                          FLU01910
      IF(NNPQ.EQ.0)GO TO 163
      WRITE(NWRITE,1071)                                                PRE00160
      DO 163 I=1,NNPQ                                                   PRE00170
      WRITE(NWRITE,1072)I,NPS(I+NNPS),Q(NCP(NPS(I+NNPS)))               PRE00180
  163 CONTINUE                                                          PRE00190
      WRITE(NWRITE,1080)                                                FLU01920
      DO 170 I=1,NNPS                                                   FLU01930
      WRITE(NWRITE,1085)I,NPS(I),T(NCP(NPS(I)))                         FLU01940
  170 CONTINUE                                                          FLU01950
  177 CONTINUE
       DO 179 I=1,MM                                                    FLU02030
        RHS(I)=0.0                                                      FLU02040
      DO 179 J=1,MM                                                     FLU02050
       TM(I,J)=0.                                                       FLU02060
  179    CONTINUE                                                       FLU02070
C                                                                       FLU02080
C    END OF  INPUT AND VERIFICATION ROUTINE                             FLU02090
C                                                                       FLU02100
      DO 300 K=1,NE                                                     FLU02110
      N1=NODE(K,1)                                                      FLU02120
      N2=NODE(K,2)                                                      FLU02130
      N3=NODE(K,3)                                                      FLU02140
       N4=NODE(K,4)                                                     FLU02150
       N5=NODE(K,5)                                                     FLU02160
       N6=NODE(K,6)                                                     FLU02170
      N7 = NODE(K,1)+NN                                                 FLU02180
      N8 =NODE(K,2)+NN                                                  FLU02190
      N9 =NODE(K,3)+NN                                                  FLU02200
      N10=NODE(K,4)+NN                                                  FLU02210
      N11=NODE(K,5)+NN                                                  FLU02220
      N12=NODE(K,6)+NN                                                  FLU02230
      N13=NCP(NODE(K,1))                                                FLU02240
      N14=NCP(NODE(K,3))                                                FLU02250
      N15=NCP(NODE(K,5))                                                FLU02260
      XC$(1)=XC(NODE(K,1))                                              FLU02270
      XC$(2)=XC(NODE(K,3))                                              FLU02280
      XC$(3)=XC(NODE(K,5))                                              FLU02290
      YC$(1)=YC(NODE(K,1))                                              FLU02300
      YC$(2)=YC(NODE(K,3))                                              FLU02310
      YC$(3)=YC(NODE(K,5))                                              FLU02320
      A=.0229248
      AA=1.                                                             FLU02340
      A1=XC$(2)*YC$(3)-XC$(3)*YC$(2)                                    FLU02350
      A2=XC$(3)*YC$(1)-XC$(1)*YC$(3)                                    FLU02360
      A3=XC$(1)*YC$(2)-XC$(2)*YC$(1)                                    FLU02370
```

```
      B1=YC$(2)-YC$(3)                                                  FLU02380
      B2=YC$(3)-YC$(1)                                                  FLU02390
      B3=YC$(1)-YC$(2)                                                  FLU02400
      C1=XC$(3)-XC$(2)                                                  FLU02410
      C2=XC$(1)-XC$(3)                                                  FLU02420
      C3=XC$(2)-XC$(1)                                                  FLU02430
      DEL=ABS(0.5*(XC$(1)*(YC$(2)-YC$(3))+XC$(2)*(YC$(3)-YC$(1))+XC$(   FLU02440
     1  3)*(YC$(1)-YC$(2))))                                            FLU02450
      CONST=(1.*A/(3.*DEL))*AA                                          FLU02460
      D1=-B1/6.                                                         FLU02470
      D2=-B2/6.                                                         FLU02480
      D3=-B3/6.                                                         FLU02490
      E1=-C1/6.                                                         FLU02500
      E2=-C2/6.                                                         FLU02510
      E3=-C3/6.                                                         FLU02520
       F1=B1/2520.                                                      FIX00010
       F2=B2/2520.                                                      FIX00020
       F3=B3/2520.                                                      FIX00030
       G1=C1/2520.                                                      FIX00040
       G2=C2/2520.                                                      FIX00050
       G3=C3/2520.                                                      FIX00060
      U1=T1(N1)                                                         FIX00080
      U2=T1(N2)                                                         FIX00090
      U3=T1(N3)                                                         FIX00100
      U4=T1(N4)                                                         FIX00110
      U5=T1(N5)                                                         FIX00120
      U6=T1(N6)                                                         FIX00130
      V1=T1(N7)                                                         FIX00140
      V2=T1(N8)                                                         FIX00150
      V3=T1(N9)                                                         FIX00160
      V4=T1(N10)                                                        FIX00170
      V5=T1(N11)                                                        FIX00180
      V6=T1(N12)                                                        FIX00190
      TM$(1,1)=.75 *(B1*B1+C1*C1)*CONST                                 FLU02590
      TM$(1,2)= (B1*B2+C1*C2)*CONST                                     FLU02600
      TM$(1,3)= -TM$(1,2)*.25                                           FLU02610
      TM$(1,4)= 0.                                                      FLU02620
      TM$(1,6)= (B1*B3+C1*C3)*CONST                                     FLU02630
      TM$(1,5)= - TM$(1,6)*.25                                          FLU02640
                 TM$(3,3)= .75 *(B2*B2+C2*C2)*CONST                     FLU02650
                 TM$(3,4)= (B2*B3+ C2*C3)*CONST                         FLU02660
                 TM$(3,5)=  -TM$(3,4)*.25                               FLU02670
      TM$(2,1)= TM$(1,2)                                                FLU02680
      TM$(2,3) = TM$(1,2)                                               FLU02690
       TM$(2,2)=8./3.*(TM$(1,1)+TM$(3,3))+2.*TM$(1,2)                   FLU02700
      TM$(2,4)= 2.*TM$(1,6) +TM$(3,4)+TM$(1,2)+4./3.*TM$(3,3)           FLU02710
      TM$(2,5)= 0.                                                      FLU02720
      TM$(2,6)= TM$(1,6)+2.*TM$(3,4) +TM$(1,2)+ 4./3.*TM$(1,1)          FLU02730
```

```
      TM$(3,1)=TM$(1,3)                                                 FLU02740
      TM$(3,2)=TM$(2,3)                                                 FLU02750
      TM$(3,6)=0.                                                       FLU02760
                    TM$(5,5)=.75 *(B3*B3+C3*C3)*CONST                   FLU02770
      TM$(4,1)=TM$(1,4)                                                 FLU02780
      TM$(4,2)=TM$(2,4)                                                 FLU02790
      TM$(4,3)= TM$(3,4)                                                FLU02800
      TM$(4,4)= 8./3.*(TM$(3,3)+TM$(5,5))+2.*TM$(3,4)                   FLU02810
      TM$(4,5)= TM$(3,4)                                                FLU02820
      TM$(4,6)= TM$(1,6)+TM$(3,4)+2.*TM$(1,2)+4./3.*TM$(5,5)            FLU02830
      TM$(6,3)=TM$(3,6)                                                 FLU02840
      TM$(5,1)= TM$(1,5)                                                FLU02850
      TM$(5,2)= TM$(2,5)                                                FLU02860
      TM$(5,3)= TM$(3,5)                                                FLU02870
      TM$(5,4)= TM$(4,5)                                                FLU02880
      TM$(5,6)= TM$(1,6)                                                FLU02890
      TM$(6,1)=TM$(1,6)                                                 FLU02900
      TM$(6,2)=TM$(2,6)                                                 FLU02910
      TM$(6,4)=TM$(4,6)                                                 FLU02920
      TM$(6,5)=TM$(5,6)                                                 FLU02930
      TM$(6,6)= 8./3.*(TM$(5,5)+TM$(1,1))+2.*TM$(1,6)                   FLU02940
       IF(NCASE.NE.1)GO TO 3000                                         FLU02950
C
C
C       BEGIN INPUT OF NON-LINEAR TERMS
C
C
       TM$(1,1)=TM$(1,1)-                                               FIX00190
     1 (F1*(78.*U1+48.*U2+(-9.*U3)+12.*U4+(-9.*U5)+48.*U6))             FIX00200
       TM$(1,2)=TM$(1,2)-                                               FIX00210
     1 (F2*(120.*U1+48.*U2-16.*U3-16.*U4-16.*U5+48.*U6))-               FIX00220
     2 (F1*(24.*U1-32.*U2-16.*U3-48.*U4+4.*U5-16.*U6))                  FIX00230
       TM$(1,3)=TM$(1,3)-                                               FIX00240
     1 (F2*(-18.*U1-32.*U2-9.*U3-20.*U4+11.*U5-16.*U6))                 FIX00250
       TM$(1,4)=TM$(1,4)-                                               FIX00260
     1 (F3*(24.*U1-32.*U2-16.*U3-48.*U4+4.*U5-16.*U6))-                 FIX00270
     2 (F2*(24.*U1-16.*U2+4.*U3-48.*U4-16.*U5-32.*U6))                  FIX00280
       TM$(1,5)=TM$(1,5)-                                               FIX00290
     1 (F3*(-18.*U1-16.*U2+11.*U3-20.*U4-9.*U5-32.*U6))                 FIX00300
       TM$(1,6)=TM$(1,6)-                                               FIX00310
     1 (F3*(120.*U1+48.*U2-16.*U3-16.*U4-16.*U5+48.*U6))-               FIX00320
     2 (F1*(24.*U1-16.*U2+4.*U3-48.*U4-16.*U5-32.*U6))                  FIX00330
       TM$(2,1)=TM$(2,1)-                                               FIX00340
     1 (F1*(48.*U1+160.*U2-32.*U3+16.*U4-20.*U5+80.*U6))                FIX00350
       TM$(2,2)=TM$(2,2)-                                               FIX00370
     1 (F2*(48.*U1+384.*U2-32.*U3+128.*U4-48.*U5+192.*U6))-             FIX00380
     2 (F1*(-32.*U1+384.*U2+48.*U3+192.*U4-48.*U5+128.*U6))             FIX00390
       TM$(2,3)=TM$(2,3)-                                               FIX00400
```

```
     1 (F2*(-32.*U1+160.*U2+48.*U3+80.*U4-20.*U5+16.*U6))                    FIX00410
       TM$(2,4)=TM$(2,4)-                                                     FIX00420
     1 (F3*(-32.*U1+384.*U2+48.*U3+192.*U4-48.*U5+128.*U6))-                  FIX00430
     2 (F2*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))                   FIX00440
       TM$(2,5)=TM$(2,5)-                                                     FIX00450
     1 (F3*(-16.*U1-96.*U2-16.*U3+16.*U4+12.*U5+16.*U6))                      FIX00460
       TM$(2,6)=TM$(2,6)-                                                     FIX00470
     1 (F3*(48.*U1+384.*U2-32.*U3+128.*U4-48.*U5+192.*U6))-                   FIX00480
     2 (F1*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))                   FIX00490
       TM$(3,1)=TM$(3,1)-                                                     FIX00500
     1 (F1*(-9.*U1-32.*U2-18.*U3-16.*U4+11.*U5-20.*U6))                       FIX00510
       TM$(3,2)=TM$(3,2)-                                                     FIX00520
     1 (F2*(-16.*U1-32.*U2+24.*U3-16.*U4+4.*U5-48.*U6))-                      FIX00530
     2 (F1*(-16.*U1+48.*U2+120.*U3+48.*U4-16.*U5-16.*U6))                     FIX00540
       TM$(3,3)=TM$(3,3)-                                                     FIX00550
     1 (F2*(-9.*U1+48.*U2+78.*U3+48.*U4-9.*U5+12.*U6))                        FIX00560
       TM$(3,4)=TM$(3,4)-                                                     FIX00570
     1 (F3*(-16.*U1+48.*U2+120.*U3+48.*U4-16.*U5-16.*U6))-                    FIX00580
     2 (F2*(4.*U1-16.*U2+24.*U3-32.*U4-16.*U5-48.*U6))                        FIX00590
       TM$(3,5)=TM$(3,5)-                                                     FIX00600
     1 (F3*(11.*U1-16.*U2-18.*U3-32.*U4-9.*U5-20.*U6))                        FIX00610
       TM$(3,6)=TM$(3,6)-                                                     FIX00620
     1 (F3*(-16.*U1-32.*U2+24.*U3-16.*U4+4.*U5-48.*U6))-                      FIX00630
     2 (F1*(4.*U1-16.*U2+24.*U3-32.*U4-16.*U5-48.*U6))                        FIX00640
       TM$(4,1)=TM$(4,1)-                                                     FIX00650
     1 (F1*(12.*U1+16.*U2-16.*U3-96.*U4-16.*U5+16.*U6))                       FIX00660
       TM$(4,2)=TM(4,2)-                                                      FIX00670
     1 (F2*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))-                  &
     2 (F1*(-48.*U1+192.*U2+48.*U3+384.*U4-32.*U5+128.*U6))                   FIX00690
       TM$(4,3)=TM$(4,3)-                                                     FIX00700
     1 (F2*(-20.*U1+80.*U2+48.*U3+160.*U4-32.*U5+16.*U6))                     FIX00710
       TM$(4,4)=TM$(4,4)-                                                     FIX00720
     1 (F3*(-48.*U1+192.*U2+48.*U3+384.*U4-32.*U5+128.*U6))-                  FIX00730
     2 (F2*(-48.*U1+128.*U2-32.*U3+384.*U4+48.*U5+192.*U6))                   FIX00740
       TM$(4,5)=TM$(4,5)-                                                     FIX00750
     1 (F3*(-20.*U1+16.*U2-32.*U3+160.*U4+48.*U5+80.*U6))                     FIX00760
       TM$(4,6)=TM$(4,6)-                                                     FIX00770
     1 (F3*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))-                  FIX00780
     2 (F1*(-48.*U1+128.*U2-32.*U3+384.*U4+48.*U5+192.*U6))                   FIX00790
       TM$(5,1)=TM$(5,1)-                                                     FIX00800
     1 (F1*(-9.*U1-20.*U2+11.*U3-16.*U4-18.*U5-32.*U6))                       FIX00810
       TM$(5,2)=TM$(5,2)-                                                     FIX00820
     1 (F2*(-16.*U1-48.*U2+4.*U3-16.*U4+24.*U5-32.*U6))-                      FIX00830
     2 (F1*(4.*U1-48.*U2-16.*U3-32.*U4+24.*U5-16.*U6))                        FIX00840
       TM$(5,3)=TM$(5,3)-                                                     FIX00850
     1 (F2*(11.*U1-20.*U2-9.*U3-32.*U4-18.*U5-16.*U6))                        FIX00860
       TM$(5,4)=TM$(5,4)-                                                     FIX00870
     1 (F3*(4.*U1-48.*U2-16.*U3-32.*U4+24.*U5-16.*U6))-                       FIX00880
```

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     2 (F2*(-16.*U1-16.*U2-16.*U3+48.*U4+120.*U5+48.*U6))            FIX00890
       TM$(5,5)=TM$(5,5)-                                               FIX00900
     1 (F3*(-9.*U1+12.*U2-9.*U3+48.*U4+78.*U5+48.*U6))                  F9X00   0
       TM$(5,6)=TM$(5,6)-                                               FIX00920
     1 (F3*(-16.*U1-48.*U2+4.*U3-16.*U4+24.*U5-32.*U6))-                FIX00930
     2 (F1*(-16.*U1-16.*U2-16.*U3+48.*U4+120.*U5+48.*U6))               FIX00940
       TM$(6,1)=TM$(6,1)-                                               FIX00950
     1 (F1*(48.*U1+80.*U2-20.*U3+16.*U4-32.*U5+160.*U6))                FIX00960
       TM$(6,2)=TM$(6,2)-                                               FIX00970
     1 (F2*(48.*U1+192.*U2-48.*U3+128.*U4-32.*U5+384.*U6))-             FIX00980
     2 (F1*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))             FIX00990
       TM$(6,3)=TM$(6,3)-                                               FIX01000
     1 (F2*(-16.*U1+16.*U2+12.*U3+16.*U4-16.*U5-96.*U6))                FIX01010
       TM$(6,4)=TM$(6,4)-                                               FIX01020
     1 (F3*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))-            FIX01030
     2 (F2*(-32.*U1+128.*U2-48.*U3+192.*U4+48.*U5+384.*U6))             FIX01040
       TM$(6,5)=TM$(6,5)-                                               FIX01050
     1 (F3*(-32.*U1+16.*U2-20.*U3+80.*U4+48.*U5+160.*U6))               FIX01060
       TM$(6,6)=TM$(6,6)-                                               FIX01070
     1 (F3*(48.*U1+192.*U2-48.*U3+128.*U4-32.*U5+384.*U6))-             FIX01080
     2 (F1*(-32.*U1+128.*U2-48.*U3+192.*U4+48.*U5+384.*U6))             FIX01090
C                                                                       FIX01100
C                                                                       FIX01110
       TM$(1,1)=TM$(1,1)-                                               FIX00200
     1(G1*(78.*V1+48.*V2+(-9.*V3)+12.*V4+(-9.*V5)+48.*V6))              FIX00210
       TM$(1,2)=TM$(1,2)-                                               FIX00220
     1(G2*(120.*V1+48.*V2-16.*V3-16.*V4-16.*V5+48.*V6))-                FIX00230
     2(G1*(24.*V1-32.*V2-16.*V3-48.*V4+4.*V5-16.*V6))                   FIX00240
       TM$(1,3)=TM$(1,3)-                                               FIX00250
     1(G2*(-18.*V1-32.*V2-9.*V3-20.*V4+11.*V5-16.*V6))                  FIX00260
       TM$(1,4)=TM$(1,4)-                                               FIX00270
     1(G3*(24.*V1-32.*V2-16.*V3-48.*V4+4.*V5-16.*V6))-                  FIX00280
     2(G2*(24.*V1-16.*V2+4.*V3-48.*V4-16.*V5-32.*V6))                   FIX00290
       TM$(1,5)=TM$(1,5)-                                               FIX00300
     1(G3*(-18.*V1-16.*V2+11.*V3-20.*V4-9.*V5-32.*V6))                  FIX00310
       TM$(1,6)=TM$(1,6)-                                               FIX00320
     1(G3*(120.*V1+48.*V2-16.*V3-16.*V4-16.*V5+48.*V6))-                FIX00330
     2(G1*(24.*V1-16.*V2+4.*V3-48.*V4-16.*V5-32.*V6))                   FIX00340
       TM$(2,1)=TM$(2,1)-                                               FIX00350
     1(G1*(48.*V1+160.*V2-32.*V3+16.*V4-20.*V5+80.*V6))                 FIX00360
                                                                        FIX00370
       TM$(2,2)=TM$(2,2)-                                               FIX00380
     1(G2*(48.*V1+384.*V2-32.*V3+128.*V4-48.*V5+192.*V6))-              FIX00390
     2(G1*(-32.*V1+384.*V2+48.*V3+192.*V4-48.*V5+128.*V6))              FIX00400
       TM$(2,3)=TM$(2,3)-                                               FIX00410
     1(G2*(-32.*V1+160.*V2+48.*V3+80.*V4-20.*V5+16.*V6))                FIX00420
       TM$(2,4)=TM$(2,4)-                                               FIX00430
     1(G3*(-32.*V1+384.*V2+48.*V3+192.*V4-48.*V5+128.*V6))-             FIX00440
```

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2(G2*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))                  FIX00450
  TM$(2,5)=TM$(2,5)-                                                    FIX00460
1(G3*(-16.*V1-96.*V2-16.*V3+16.*V4+12.*V5+16.*V6))                     FIX00470
  TM$(2,6)=TM$(2,6)-                                                    FIX00480
1(G3*(48.*V1+384.*V2-32.*V3+128.*V4-48.*V5+192.*V6))-                  FIX00490
2(G1*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))                  FIX00500
  TM$(3,1)=TM$(3,1)-                                                    FIX00510
1(G1*(-9.*V1-32.*V2-18.*V3-16.*V4+11.*V5-20.*V6))                      FIX00520
  TM$(3,2)=TM$(3,2)-                                                    FIX00530
1(G2*(-16.*V1-32.*V2+24.*V3-16.*V4+4.*V5-48.*V6))-                     FIX00540
2(G1*(-16.*V1+48.*V2+120.*V3+48.*V4-16.*V5-16.*V6))                    FIX00550
  TM$(3,3)=TM$(3,3)-                                                    FIX00560
1(G2*(-9.*V1+48.*V2+78.*V3+48.*V4-9.*V5+12.*V6))                       FIX00570
  TM$(3,4)=TM$(3,4)-                                                    FIX00580
1(G3*(-16.*V1+48.*V2+120.*V3+48.*V4-16.*V5-16.*V6))-                   FIX00590
2(G2*(4.*V1-16.*V2+24.*V3-32.*V4-16.*V5-48.*V6))                       FIX00600
  TM$(3,5)=TM$(3,5)-                                                    FIX00610
1(G3*(11.*V1-16.*V2-18.*V3-32.*V4-9.*V5-20.*V6))                       FIX00620
  TM$(3,6)=TM$(3,6)-                                                    FIX00630
1(G3*(-16.*V1-32.*V2+24.*V3-16.*V4+4.*V5-48.*V6))-                     FIX00640
2(G1*(4.*V1-16.*V2+24.*V3-32.*V4-16.*V5-48.*V6))                       FIX00650
  TM$(4,1)=TM$(4,1)-                                                    FIX00660
1(G1*(12.*V1+16.*V2-16.*V3-96.*V4-16.*V5+16.*V6))                      FIX00670
  TM$(4,2)=TM(4,2)-                                                     FIX00680
1(G2*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))-                 FIX00690
2(G1*(-48.*V1+192.*V2+48.*V3+384.*V4-32.*V5+128.*V6))                  FIX00700
  TM$(4,3)=TM$(4,3)-                                                    FIX00710
1(G2*(-20.*V1+80.*V2+48.*V3+160.*V4-32.*V5+16.*V6))                    FIX00720
  TM$(4,4)=TM$(4,4)-                                                    FIX00730
1(G3*(-48.*V1+192.*V2+48.*V3+384.*V4-32.*V5+128.*V6))-                 FIX00740
2(G2*(-48.*V1+128.*V2-32.*V3+384.*V4+48.*V5+192.*V6))                  FIX00750
  TM$(4,5)=TM$(4,5)-                                                    FIX00760
1(G3*(-20.*V1+16.*V2-32.*V3+160.*V4+48.*V5+80.*V6))                    FIX00770
  TM$(4,6)=TM$(4,6)-                                                    FIX00780
1(G3*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))-                 FIX00790
2(G1*(-48.*V1+128.*V2-32.*V3+384.*V4+48.*V5+192.*V6))                  FIX00800
  TM$(5,1)=TM$(5,1)-                                                    FIX00810
1(G1*(-9.*V1-20.*V2+11.*V3-16.*V4-18.*V5-32.*V6))                      FIX00820
  TM$(5,2)=TM$(5,2)-                                                    FIX00830
1(G2*(-16.*V1-48.*V2+4.*V3-16.*V4+24.*V5-32.*V6))-                     FIX00840
2(G1*(4.*V1-48.*V2-16.*V3-32.*V4+24.*V5-16.*V6))                       FIX00850
  TM$(5,3)=TM$(5,3)-                                                    FIX00860
1(G2*(11.*V1-20.*V2-9.*V3-32.*V4-18.*V5-16.*V6))                       FIX00870
  TM$(5,4)=TM$(5,4)-                                                    FIX00880
1(G3*(4.*V1-48.*V2-16.*V3-32.*V4+24.*V5-16.*V6))-                      FIX00890
2(G2*(-16.*V1-16.*V2-16.*V3+48.*V4+120.*V5+48.*V6))                    FIX00900
  TM$(5,5)=TM$(5,5)-                                                    FIX00910
1(G3*(-9.*V1+12.*V2-9.*V3+48.*V4+78.*V5+48.*V6))                       FIX00920
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      TM$(5,6)=TM$(5,6)-                                               FIX00930
     1(G3*(-16.*V1-48.*V2+4.*V3-16.*V4+24.*V5-32.*V6))-                FIX00940
     2(G1*(-16.*V1-16.*V2-16.*V3+48.*V4+120.*V5+48.*V6))               FIX00950
      TM$(6,1)=TM$(6,1)-                                               FIX00960
     1(G1*(48.*V1+80.*V2-20.*V3+16.*V4-32.*V5+160.*V6))                FIX00970
      TM$(6,2)=TM$(6,2)-                                               FIX00980
     1(G2*(48.*V1+192.*V2-48.*V3+128.*V4-32.*V5+384.*V6))-             697   99
     2(G1*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))             FIX01000
      TM$(6,3)=TM$(6,3)-                                               FIX01010
     1(G2*(-16.*V1+16.*V2+12.*V3+16.*V4-16.*V5-96.*V6))                FIX01020
      TM$(6,4)=TM$(6,4)-                                               FIX01030
     1(G3*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))-            FIX01040
     2(G2*(-32.*V1+128.*V2-48.*V3+192.*V4+48.*V5+384.*V6))             FIX01050
      TM$(6,5)=TM$(6,5)-                                               FIX01060
     1(G3*(-32.*V1+16.*V2-20.*V3+80.*V4+48.*V5+160.*V6))               FIX01070
      TM$(6,6)=TM$(6,6)-                                               FIX01080
     1(G3*(48.*V1+192.*V2-48.*V3+128.*V4-32.*V5+384.*V6))-             FIX01090
     2(G1*(-32.*V1+128.*V2-48.*V3+192.*V4+48.*V5+384.*V6))             FIX01100
C                                                                      FIX01110
C                                                                      FIX01120
C           THIS ENDS THE ADDITION OF NON-LINEAR TERMS TO              FIX01120
C                 THE LOCAL ARRAY.                                     FIX01140
C                                                                      FIX01150
 3000 CONTINUE                                                         FLU03380
     TM$(7,7)=TM$(1,1)                                                 FLU03390
     TM$(7,8)=TM$(1,2)                                                 FLU03400
     TM$(7,9)=TM$(1,3)                                                 FLU03410
     TM$(7,10)=TM$(1,4)                                                FLU03420
     TM$(7,11)=TM$(1,5)                                                FLU03430
     TM$(7,12)=TM$(1,6)                                                FLU03440
     TM$(8,7)=TM$(2,1)                                                 FLU03450
     TM$(8,8)=TM$(2,2)                                                 FLU03460
     TM$(8,9)=TM$(2,3)                                                 FLU03470
     TM$(8,10)=TM$(2,4)                                                FLU03480
     TM$(8,11)=TM$(2,5)                                                FLU03490
     TM$(8,12)=TM$(2,6)                                                FLU03500
     TM$(9,7)=TM$(3,1)                                                 FLU03510
     TM$(9,8)=TM$(3,2)                                                 FLU03520
     TM$(9,9)=TM$(3,3)                                                 FLU03530
     TM$(9,10)=TM$(3,4)                                                FLU03540
     TM$(9,11)=TM$(3,5)                                                FLU03550
     TM$(9,12)=TM$(3,6)                                                FLU03560
     TM$(10,7)=TM$(4,1)                                                FLU03570
     TM$(10,8)=TM$(4,2)                                                FLU03580
     TM$(10,9)=TM$(4,3)                                                FLU03590
     TM$(10,10)=TM$(4,4)                                               FLU03600
     TM$(10,11)=TM$(4,5)                                               FLU03610
     TM$(10,12)=TM$(4,6)                                               FLU03620
```

```
TM$(11,7)=TM$(5,1)                                              FLU03630
TM$(11,8)=TM$(5,2)                                              FLU03640
TM$(11,9)=TM$(5,3)                                              FLU03650
TM$(11,10)=TM$(5,4)                                             FLU03660
TM$(11,11)=TM$(5,5)                                             FLU03670
TM$(11,12)=TM$(5,6)                                             FLU03680
TM$(12,7)=TM$(6,1)                                              FLU03690
TM$(12,8)=TM$(6,2)                                              FLU03700
TM$(12,9)=TM$(6,3)                                              FLU03710
TM$(12,10)=TM$(6,4)                                             FLU03720
TM$(12,11)=TM$(6,5)                                             FLU03730
TM$(12,12)=TM$(6,6)                                             FLU03740
TM$(13,1)= D1                                                   FLU03750
TM$(1,13)=TM$(13,1)                                             FLU03760
TM$(14,1)=0.                                                    FLU03770
TM$(1,14)=TM$(14,1)                                             FLU03780
TM$(15,1)=0.                                                    FLU03790
TM$(1,15)=TM$(15,1)                                             FLU03800
TM$(13,2)= D1+ 2.*D2                                            FLU03810
TM$(2,13)=TM$(13,2)                                             FLU03820
TM$(14,2)=2.*D1+D2                                              FLU03830
TM$(2,14)=TM$(14,2)                                             FLU03840
TM$(15,2)= D1+D2                                                FLU03850
TM$(2,15)=TM$(15,2)                                             FLU03860
TM$(13,3)= 0.                                                   FLU03870
TM$(3,13)=TM$(13,3)                                             FLU03880
TM$(14,3)=D2                                                    FLU03890
TM$(3,14)=TM$(14,3)                                             FLU03900
TM$(15,3)= 0.                                                   FLU03910
TM$(3,15)=TM$(15,3)                                             FLU03920
TM$(13,4)=D2+D3                                                 FLU03930
TM$(4,13)=TM$(13,4)                                             FLU03940
TM$(14,4)=D2+2.*D3                                              FLU03950
TM$(4,14)=TM$(14,4)                                             FLU03960
TM$(15,4)=2.*D2+D3                                              FLU03970
TM$(4,15)=TM$(15,4)                                             FLU03980
TM$(13,5)=0.                                                    FLU03990
TM$(5,13)=TM$(13,5)                                             FLU04000
TM$(14,5)=0.                                                    FLU04010
TM$(5,14)=TM$(14,5)                                             FLU04020
TM$(15,5)=D3                                                    FLU04030
TM$(5,15)=TM$(15,5)                                             FLU04040
TM$(13,6)=D1+2.*D3                                              FLU04050
TM$(6,13)=TM$(13,6)                                             FLU04060
TM$(14,6)=D1+D3                                                 FLU04070
TM$(6,14)=TM$(14,6)                                             FLU04080
TM$(15,6)=2.*D1+D3                                              FLU04090
TM$(6,15)=TM$(15,6)                                             FLU04100
```

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      TM$(13,7)=E1                                                    FLU04110
      TM$(7,13)=TM$(13,7)                                             FLU04120
      TM$(14,7)=0.                                                    FLU04130
      TM$(7,14)=TM$(14,7)                                             FLU04140
      TM$(15,7)=0.                                                    FLU04150
      TM$(7,15)=TM$(15,7)                                             FLU04160
      TM$(13,8)=E1+2.*E2                                              FLU04170
      TM$(8,13)=TM$(13,8)                                             FLU04180
      TM$(14,8)=2.*E1+E2                                              FLU04190
      TM$(8,14)=TM$(14,8)                                             FLU04200
      TM$(15,8)= E1+E2                                                FLU04210
      TM$(8,15)=TM$(15,8)                                             FLU04220
      TM$(13,9)= 0.                                                   FLU04230
      TM$(9,13)=TM$(13,9)                                             FLU04240
      TM$(14,9)= E2                                                   FLU04250
      TM$(9,14)=TM$(14,9)                                             FLU04260
      TM$(15,9)= 0.                                                   FLU04270
      TM$(9,15)=TM$(15,9)                                             FLU04280
      TM$(13,10)=E2+E3                                                FLU04290
      TM$(10,13)=TM$(13,10)                                           FLU04300
      TM$(14,10)=E2+2.*E3                                             FLU04310
       TM$(10,14)=TM$(14,10)                                          FLU04320
        TM$(15,10)=E2+E2+E3                                           FLU04330
      TM$(10,15)=TM$(15,10)                                           FLU04340
      TM$(13,11)= 0.                                                  FLU04350
      TM$(11,13)=TM$(13,11)                                           FLU04360
      TM$(14,11)=0.                                                   FLU04370
      TM$(11,14)=TM$(14,11)                                           FLU04380
      TM$(15,11)= E3                                                  FLU04390
      TM$(11,15)=TM$(15,11)                                           FLU04400
         TM$(13,12)=E1+E3+E3                                          FLU04410
      TM$(12,13)=TM$(13,12)                                           FLU04420
      TM$(14,12)=E1+E3                                                FLU04430
      TM$(12,14)=TM$(14,12)                                           FLU04440
      TM$(15,12)=2.*E1+E3                                             FLU04450
      TM$(12,15)=TM$(15,12)                                           FLU04460
C
       CONST2=1/(4.*DEL)
      TM$(13,13)=-.5*(B1*B1+C1*C1) *CONST2
      TM$(13,14)=-.5*(B1*B2+C1*C2)*CONST2
      TM$(13,15)=-.5*(B1*B3+C1*C3)*CONST2
      TM$(14,13)=TM$(13,14)
      TM$(14,14)=-.5*(B2*B2+C2*C2)*CONST2
      TM$(14,15)=-.5*(B2*B3+C2*C3)*CONST2
      TM$(15,13)=TM$(13,15)
      TM$(15,14)=TM$(14,15)
      TM$(15,15)=-.5*(B3*B3+C3*C3)*CONST2
C                                                                     QZ 00170
```

```
C                                                                        QZ  00180
      U(1)=U1                                                            QZ  00190
      U(2)=U2                                                            QZ  00200
      U(3)=U3                                                            QZ  00210
      U(4)=U4                                                            QZ  00220
      U(5)=U5                                                            QZ  00230
      U(6)=U6                                                            QZ  00240
      V(1)=V1                                                            QZ  00250
      V(2)=V2                                                            QZ  00260
      V(3)=V3                                                            QZ  00270
      V(4)=V4                                                            QZ  00280
      V(5)=V5                                                            QZ  00290
      V(6)=V6                                                            QZ  00300
      NT(1)=N1
      NT(2)=N3
      NT(3)=N5
C                                                                        QZ  00340
C                                                                        QZ  00350
C                                                                        QZ  00360
      CONST3=1./DEL*4.*60.
      QZ1(1,1)=0.0                                                       QAR00030
      QZ1(2,1)=56.*(B2*C1-B1*C2)*CONST3                                  QAR00040
      QZ1(2,2)=0.0                                                       QAR00050
      QZ1(3,1)=-8.*(B2*C1-B1*C2)*CONST3                                  QAR00060
      QZ1(3,2)=12.*(B2*C1-B1*C2)*CONST3                                  QAR00070
      QZ1(3,3)=0.0                                                       QAR00080
      QZ1(4,1)=12.*(B3*C1+B2*C1-B1*C3-B1*C2)*CONST3                      QAR00090
      QZ1(4,2)=(16.*(B2*C1-B1*C2)+32.*(B3*C2+B3*C1-B2*C3-B1*C3))*CONST3  QAR00100
      QZ1(4,3)=12.*(B3*C2-B2*C3)*CONST3                                  QAR00110
      QZ1(4,4)=0.0                                                       QAR00120
      QZ1(5,1)=-8.*(B3*C1-B1*C3)*CONST3                                  QAR00130
      QZ1(5,2)=(-8.*(B3*C2-B2*C3)-4.*(B3*C1-B1*C3))*CONST3               QAR00140
      QZ1(5,3)=-4.*(B3*C2-B2*C3)*CONST3                                  QAR00150
      QZ1(5,4)=12.*(B3*C2-B2*C3)*CONST3                                  QAR00160
      QZ1(5,5)=0.0                                                       QAR00170
      QZ1(6,1)=56.*(B3*C1-B1*C3)*CONST3                                  QAR00180
      QZ1(6,2)=(96.*(B3*C2-B2*C3)+32.*(B1*C2+B3*C1-B2*C1-B1*C3))*CONST3  QAR00190
      QZ1(6,3)=(-4.*(B1*C2-B2*C1)-8.*(B3*C2-B2*C3))*CONST3               QAR00200
      QZ1(6,4)=(16.*(B1*C3-B3*C1)+32.*(B3*C2+B1*C2-B2*C3-B2*C1))*CONST3  QAR00210
      QZ1(6,5)=12.*(B1*C3-B3*C1)*CONST3                                  QAR00220
      QZ1(6,6)=0.0                                                       QAR00230
      QZ2(1,1)=0.0                                                       QAR00030
      QZ2(2,1)=12.*(B2*C1-B1*C2)*CONST3                                  QAR00040
      QZ2(2,2)=0.0                                                       QAR00050
      QZ2(3,1)=-8.*(B2*C1-B1*C2)*CONST3                                  QAR00060
      QZ2(3,2)=56.*(B2*C1-B1*C2)*CONST3                                  QAR00070
      QZ2(3,3)=0.0                                                       QAR00080
      QZ2(4,1)=(-8.*(B3*C1-B1*C3)-4.*(B2*C1-B1*C2))*CONST3               QAR00090
```

```
      QZ2(4,2)=(96.*(B3*C1-B1*C3)+32.*(B2*C1+B3*C2-B1*C2-B2*C3))*CONST3  QAR00100
      QZ2(4,3)=56.*(B3*C2-B2*C3)*CONST3                                   QAR00110
      QZ2(4,4)=0.0                                                        QAR00120
      QZ2(5,1)=-4.*(B3*C1-B1*C3)*CONST3                                   QAR00130
      QZ2(5,2)=(-4.*(B3*C2-B2*C3)-8.*(B3*C1-B1*C3))*CONST3                QAR00140
      QZ2(5,3)=-8.*(B3*C2-B2*C3)*CONST3                                   QAR00150
      QZ2(5,4)=12.*(B3*C2-B2*C3)*CONST3                                   QAR00160
      QZ2(5,5)=0.0                                                        QAR00170
      QZ2(6,1)=12.*(B3*C1-B1*C3)*CONST3                                   QAR00180
      QZ2(6,2)=(16.*(B1*C2-B2*C1)+32.*(B3*C1+B3*C2-B1*C3-B2*C3))*CONST3  QAR00190
      QZ2(6,3)=12.*(B1*C2+B3*C2-B2*C1-B2*C3)*CONST3                       QAR00200
      QZ2(6,4)=(16.*(B3*C2-B2*C3)+32.*(B1*C3+B1*C2-B3*C1-B2*C1))*CONST3  QAR00210
      QZ2(6,5)=12.*(B1*C3-B3*C1)*CONST3                                   QAR00220
      QZ2(6,6)=0.0                                                        QAR00230
      QZ3(1,1)=0.0                                                        QAR00030
      QZ3(2,1)=12.*(B2*C1-B1*C2)*CONST3                                   QAR00040
      QZ3(2,2)=0.0                                                        QAR00050
      QZ3(3,1)=-4.*(B2*C1-B1*C2)*CONST3                                   QAR00060
      QZ3(3,2)=12.*(B2*C1-B1*C2)*CONST3                                   QAR00070
      QZ3(3,3)=0.0                                                        QAR00080
      QZ3(4,1)=(-4.*(B3*C1-B1*C3)-8.*(B2*C1-B1*C2))*CONST3                QAR00090
       QZ3(4,2)=(16.*(B3*C2-B2*C3)+32.*(B2*C1+B3*C1-B1*C2-B1*C3))*CONST3QAR00100
       QZ3(4,3)=12.*(B3*C2-B2*C3)*CONST3                                  QAR00110
      QZ3(4,4)=0.0                                                        QAR00120
      QZ3(5,1)=-8.*(B3*C1-B1*C3)*CONST3                                   QAR00130
      QZ3(5,2)=(12.*(B3*C2-B2*C3+B3*C1-B1*C3))*CONST3                     QAR00140
      QZ3(5,3)=-8.*(B3*C2-B2*C3)*CONST3                                   QAR00150
      QZ3(5,4)=56.*(B3*C2-B2*C3)*CONST3                                   QAR00160
      QZ3(5,5)=0.0                                                        QAR00170
      QZ3(6,1)=12.*(B3*C1-B1*C3)*CONST3                                   QAR00180
      QZ3(6,2)=(16.*(B3*C1-B1*C3)+32.*(B1*C2+B3*C2-B2*C1-B2*C3))*CONST3  QAR00190
      QZ3(6,3)=(-8.*(B1*C2-B2*C1)-4.*(B3*C2-B2*C3))*CONST3                QAR00200
      QZ3(6,4)=(96.*(B1*C2-B2*C1)+32.*(B3*C2+B1*C3-B2*C3-B3*C1))*CONST3
      QZ3(6,5)=56.*(B1*C3-B3*C1)*CONST3                                   QAR00220
      QZ3(6,6)=0.0                                                        QAR00230
C                                                                         QZ 00380
C                                                                         QZ 00390
       DO 57  JA=1,14
       DO 57  JB=1,3                                                      QZ 00410
       ICHECK=NT(JB)-NI(JA)
       IF(ICHECK.EQ.0) GO TO 58
       GO TO 57
   58  IF(JB-2)60,70,80
   60 QPRM=0.0                                                            QZ 00470
       DO 61  KO=1,6                                                      QZ 00480
       DO 61  KM=1,KO                                                     QZ 00490
      QPRM=QPRM+(U(KO)*V(KM)-U(KM)*V(KO))*QZ1(KO,KM)                      QZ 00510
   61 CONTINUE                                                            QZ 00520
```

```
      Q(NCP(NT(JB)))=QPRM+Q(NCP(NT(JB)))
       GO TO 57
 70    QPRM=0.0
       DO 71  KN=1,6
       DO 71  KL=1,KN
      QPRM=QPRM+(U(KN)*V(KL)-U(KL)*V(KN))*QZ2(KN,KL)
 71    CONTINUE
      Q(NCP(NT(JB)))=QPRM+Q(NCP(NT(JB)))
       GO TO 57
 80   QPRM=0.0
       DO 81  KP=1,6
       DO 81  KQ=1,6
       QPRM=QPRM+(U(KP)*V(KQ)-U(KQ)*V(KP))*QZ3(KP,KQ)
 81   CONTINUE
      Q(NCP(NT(JB)))=QPRM+Q(NCP(NT(JB)))
 57   CONTINUE
      QZC(1)=C1*C1
      QZC(2)=2.*C1*C2
      QZC(3)=C2*C2
      QZC(4)=2.*C2*C3
      QZC(5)=C3*C3
      QZC(6)=2.*C1*C3
      QZB(1)=B1*B1
      QZB(2)=2.*B1*B2
      QZB(3)=B2*B2
      QZB(4)=2.*B2*B3
      QZB(5)=B3*B3
      QZB(6)=2.*C1*C3
      NLCY(1)=17
      NLCY(2)=18
      NLCY(3)=20
      NLCY(4)=22
      NRCY(1)=30
      NRCY(2)=33
      NRCY(3)=34
      NRCY(4)=36
      CONST4=A/(4.*DEL**2)
      DO 513 LA=1,4
      JCHECK=K-NLCY(LA)
      KCHECK=K-NRCY(LA)
      IF(JCHECK.EQ.0) GO TO 517
      IF(KCHECK.EQ.0) GO TO 519
      GO TO 513
517   THETA=ATAN((YC(NT(3))-YC(NT(2)))/(XC(NT(3))-XC(NT(2))))
      THETA1=THETA-2.*ATAN(1.)
      SLIJ=SQRT((YC(NT(3))-YC(NT(2)))**2+(XC(NT(3))-XC(NT(2)))**2)
      XHAT=COS(THETA1)
      YHAT=SIN(THETA1)
```

```
      PART1=0.0
      PART2=0.0
      TPART=0.0
      DO 523 LB=1,6
      PART1=PART1+(U(LB)*QZB(LB))*CONST4
      PART2=PART2+(V(LB)*QZC(LB))*CONST4
  523 CONTINUE
      TPART=PART1+PART2
      Q(NCP(NT(2)))=Q(NCP(NT(2)))+TPART*SLIJ
      Q(NCP(NT(3)))=Q(NCP(NT(3)))+TPART*SLIJ
      GC TO 513
  519 THETA2=ATAN((YC(NT(1))-YC(NT(2)))/(XC(NT(2))-XC(NT(1))))         GO 00210
      THETA1=6.*ATAN(1.)-THETA2                                          GO 00220
      SLIJ=SQRT((YC(NT(1))-YC(NT(2)))**2+(XC(NT(1))-XC(NT(2)))**2)
      XHAT=COS(THETA1)                                                   GO 00240
      YHAT=SIN(THETA1)                                                   GO 00250
      TPART=0.0
      PART1=0.0
      PART2=0.0
      DO 521 LB=1,6                                                      GO 00260
      PART1=PART1+(U(LB)*QZB(LB))*CONST4                                 GO 00270
      PART2=PART2+(V(LB)*QZC(LB))*CONST4                                 GO 00280
  521 CONTINUE                                                           GO 00290
      TPART=PART1+PART2                                                  GO 00300
      Q(NCP(NT(1)))=Q(NCP(NT(1)))+TPART*SLIJ
      Q(NCP(NT(2)))=Q(NCP(NT(2)))+TPART*SLIJ
  513 CONTINUE                                                           GO 00320
  185 CCNTINUE                                                           FLU04470
      N(1)= N1                                                           FLU04480
      N(2)= N2                                                           FLU04490
      N(3)= N3                                                           FLU04500
       N(4)=N4                                                           FLU04510
       N(5)=N5                                                           FLU04520
       N(6)=N6                                                           FLU04530
      N(7) =N7                                                           FLU04540
      N(8) =N8                                                           FLU04550
      N(9) =N9                                                           FLU04560
      N(10)=N10                                                          FLU04570
      N(11)=N11                                                          FLU04580
      N(12)=N12                                                          FLU04590
      N(13)=N13                                                          FLU04600
      N(14)=N14                                                          FLU04610
      N(15)=N15                                                          FLU04620
      DO 200  I$=1,15                                                    FLU04630
      I= N(I$)                                                           FLU04640
      DO 200  J$=1,15                                                    FLU04650
      J=N(J$)                                                            FLU04660
      TM(I,J)=TM(I,J)+TM$(I$,J$)                                         FLU04670
```

```
  200 CONTINUE                                                              FLU04680
  300 CONTINUE                                                              FLU04690
      IF(NNQXY.EQ.0) GO TO 310                                              FLU04700
      DO 310  I=1,NNQXY                                                     FLU04710
      RHS(NQS(I))=RHS(NQS(I))+Q(NQS(I))                                     FLU04720
        RHS(NQS(I)+NN)=RHS(NQS(I)+NN)+Q(NQS(I)+NN)                          FLU04730
      IF(I-NNPQ)309,309,310
  309 RHS(NCP(NPS(I+NNPS)))=RHS(NCP(NPS(I+NNPS)))+Q(NCP(NPS(I+NNPS)))
  310 CONTINUE                                                              FLU04740
  320 CONTINUE                                                              FLU04750
      CALL SOLMIX(MM,TM,T,RHS,NTOTVP,NVIS,NTOTQ,NQIS,B,Z,C)                 FLU04760
      WRITE(NWRITE,2000)                                                    FLU04770
      NTEST=0
       DO 322 J=1,MM                                                        FLU04780
       ESPILA=ABS(T1(J)-T(J))                                               FLU04790
       IF(ESPILA-.001)322,324,324                                           FLU04800
  322 CONTINUE                                                              FLU04810
      NTEST=100
  324 CONTINUE                                                              FLU04820
      DO 325 I=1,NN
       IV=I+NN
       IQ=I+2*NN
       T1(I)=             T(I)
       Q(I)=RHS(I)
       T1(IV)=              T(IV)
       Q(IV)=RHS(IV)
       IF(MM-IQ)2223,2222,2222
 2222  T1(IQ)=              T(IQ)
       Q(IQ)=RHS(IQ)
       WRITE(NWRITE,2300)I,T(I),IV,T(IV),IQ,T(IQ),Q(I),Q(IV),Q(IQ)
       GO TO 325
 2223  WRITE(NWRITE,2301)I,T(I),IV,T(IV),Q(I),Q(IV)
  325 CONTINUE                                                              FLU04860
      IF(NTEST.NE.100) GO TO 177
  500 FORMAT(I10)                                                           FLU04880
 1005 FORMAT(3I10)                                                          FLU04890
 1006 FORMAT(6X,A4,I10,2F10.0)                                              FLU04900
 1010 FORMAT(7I10)                                                          FLU04910
 1015 FORMAT(6X,A4,I10,F10.0)                                               FLU04920
 1016 FORMAT(6X,A4,I10)                                                     FLU04930
 1020 FORMAT(I10,2F10.0)                                                    FLU04940
 1025 FORMAT(6X,A4,I10,F10.0)                                               FLU04950
 1030 FORMAT(6X,A4,2I10,2F10.0)                                             FLU04960
 1035 FORMAT(5X,'NO. OF NODES=',I3,//,5X,'NO. OF ELEMENTS=',I3,//,          FLU04970
     15X,'NO. OF CORNER NODES=',I3,//)                                      FLU04980
 1040 FORMAT(5X,'SUMMARY OF NODAL COORDINATES',//,                          FLU04990
     17X,'I',12X'X(I)',12X,'Y(I)',//)                                       FLU05000
 1045 FORMAT(5X,I3,2(7X,F10.3))                                             FLU05010
```

```
1050 FORMAT(5X,'LISTING OF SYSTEM TOPOLOGY'//,5X                          FLU05020
    1'ELEMENT NUMBER',20X,'NODE NUMBERS',//)                            FLU05030
1055 FORMAT(5X,I3,10X,6(5X,I3))                                           FLU05040
1060 FORMAT(7X,'NODES WHERE VELOCITIES ARE SPECIFIED',                    FLU05050
    1//,8X,'I',5X,'NODE',5X,'U VELOCITY',5X,'V VELOCITY',//)           FLU05060
1065 FORMAT(2X,2(4X,I3),3X,F12.3,3X,F12.3)                                FLU05070
1070 FORMAT(5X,'SPECIFIED RIGHT HAND SIDE I.E. QX AND QY',                FLU05080
    1      //,2X,'                NODE',5X,'         QX',5X,'    QY',//)  FLU05090
1071 FORMAT(5X,'SPECIFIED RIGHT HAND SIDE FOR PRESSURE NODES, I.E. QZ'   PRE00200
    1,//,2X,'               NODE',5X,'              QZ',//)              PRE00210
1072 FORMAT(2X,2(4X,I3),3X,F12.3)                                         PRE00220
1075 FORMAT(5X,I3,2(10X,F12.3))                                           FLU05100
1080 FORMAT(5X,'                NODES WHERE PRESSURE IS SPECIFIED',       FLU05110
    1              //,5X,'I',5X,'NODE',10X,'PRESSURE',//)               FLU05120
1085 FORMAT(5X,I3,3X,I3,10X,F12.3)                                        FLU05130
2300 FORMAT(6X,3(I3,5X,E13.5,5X),3(E13.5,3X))
2301 FORMAT(6X,2(I3,5X,E13.5,5X),26X,2(E13.5,3X))
2000 FORMAT(3X,'NODE NO.',3X,'T(J)  X-COORD',5X,'NODE NO.',2X,'T(J)',2
    1X,'Y-COORD',5X,'NODE NO.',4X,'T(I)',9X,'Q(I) X',17X,'Q(I) Y',12
    2 X,'Q(I) P')
2020 FORMAT(1H1,5X,'THIS IS A NON-LINEAR PROBLEM',//)                     FLU05210
     STOP                                                                FLU05220
     END                                                                 FLU05230
```

```
      SUBROUTINE SOLMIX(N,A,X,Y,NX,LISTX,NY,LISTY,B,Z,C)
      DIMENSION A(N,N),X(N),Y(N),LISTX(N),LISTY(N)
      DIMENSION  B(NY,NY),Z(NY),C(NY)
      IF(N.EQ.(NX+NY)) GO TO 100
      WRITE(6,111) N,NX,NY
  111 FORMAT(10X,'N=',I2,10X,'NX=',I2,10X,'NY=',I2)
      STOP
  100 CONTINUE
      IF(NX.NE.0) GO TO 120
      CALL SOLVE(N,A,Y,X)
      RETURN
  120 CONTINUE
      IF(NY.NE.0)  GO TO 140
      DO 130  I=1,N
      Y(I)= 0.
      DO 130 J= 1,N
  130 Y(I)=Y(I) +A(I,J)*X(J)
      RETURN
  140 CONTINUE
      DO 200 I= 1,NY
      DO 200 J= 1,NY
  200 B(I,J)=A(LISTY(I),LISTY(J))
      DO 300  I= 1,NY
      C(I) = Y(LISTY(I))
      DO 300  J=1,NX
  300 C(I)= C(I)-A(LISTY(I),LISTX(J))*X(LISTX(J))
      CALL SOLVE(NY,B,C,Z)
      DO 400 I=1,NY
  400 X(LISTY(I))= Z(I)
      DO 800 I= 1,NX
      C(I)=0.
      DO 600  J= 1,NY
  600 C(I)=C(I) +A(LISTX(I),LISTY(J))*X(LISTY(J))
      DO 700 J= 1,NX
  700 C(I) =C(I)+A(LISTX(I),LISTX(J))*X(LISTX(J))
  800 Y(LISTX(I))= C(I)
      RETURN
      END
```

```
      SUBROUTINE SOLVE(N,A,C,X)                                         SOL00010
       DATA NWRITE/6/
      DIMENSION A(N,N),X(N),C(N)                                        SOL00020
      DO 400  K= 1,N                                                    SOL00030
      BIG = ABS(A(K,K))                                                 SOL00040
      IBIG = K                                                          SOL00050
      DO 100    I=K,N                                                   SOL00060
      SIZE = ABS(A(I,K))                                                SOL00070
      IF(SIZE.LT.BIG)  GO TO 100                                        SOL00080
      BIG=SIZE                                                          SOL00090
      IBIG =I                                                           SOL00100
  100 CONTINUE                                                          SOL00110
      IF(K.EQ.IBIG) GO TO 280                                           SOL00120
      DO 200   J= K,N                                                   SOL00130
      ABIG = A(IBIG,J)                                                  SOL00140
      A(IBIG,J)=A(K,J)                                                  SOL00150
  200 A(K,J)= ABIG                                                      SOL00160
      CBIG =C(IBIG)                                                     SOL00170
      C(IBIG) =C(K)                                                     SOL00180
      C(K) =CBIG                                                        SOL00190
  280 CONTINUE                                                          SOL00200
      IF(A(K,K).EQ.0.) GO TO 600                                        SOL00210
      DO 350  I= 1,N                                                    SOL00220
      IF(I.EQ.K) GO TO 350                                              SOL00230
      RATIO = A(I,K)/A(K,K)                                             SOL00240
      DO 300  J=K,N                                                     SOL00250
      A(I,J)=A(I,J)-RATIO*A(K,J)                                        SOL00260
      CALL OVERFL(IQ)
      IF(IQ.EQ.2)GO TO 301
      WRIRE(6,98)
   98 FORMAT(1X,'IQ I       J    K  A(I,J)     A(K,J)                 RATIO')
      WRITE(6,99)IQ,I,J,K,A(I,J),A(K,J),RATIO
   99 FORMAT(1X,4I5,3E15.6)
C     IF IQ= 1 OVERFLOW......IQ=3  UNDERFLOW
  301 CONTINUE
  300 CONTINUE
      C(I) = C(I) -RATIO*C(K)                                           SOL00270
  350 CONTINUE                                                          SOL00280
  400 CONTINUE                                                          SOL00290
      DO 500 K= 1,N                                                     SOL00300
  500 X(K) = C(K)/A(K,K)                                                SOL00310
      RETURN                                                            SOL00320
  600 WRITE(6,666)                                                      SOL00330
  666 FORMAT(10X,'SINGULAR MATRIX')                                     SOL00340
      DO 700 I= 1,N                                                     SOL00350
  700 X(I)= 0.                                                          SOL00360
      WRITE(NWRITE,2)K
    2 FORMAT(6X,'K=',I5)
```

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